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INFLUENCE OF LOCAL POST-BUCKLING BEHAVIOUR ON BENDING OF THIN-WALLED ELASTIC BEAMS WITH CENTRAL INTERMEDIATE STIFFENERS

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The influence of the local post-buckling behaviour on bending of thin-walled beams is studied. A "lower bound" approach by KOITER and PIGNATARO [1] enables to determine the overall flexural stiffness of a beam after its local buckling. The results obtained are compared with data reported by other authors.

1. INTRODUCTION

Thin-walled structures, especially columns and beams, are able to work after local buckling; the determination of their load carrying capacity requires considering the interaction of buckling modes and imperfections in the nonlinear analysis of stability. Since pre-buckling stresses in the cross-section vary linearly, the linear problem of beam bending is more complex than that of the thin-walled columns under compression, as far as local buckling is considered. The nonlinear problem of bending is more simple as only the effect of local buckling on the global bending has to be taken into account. "Reduced" flexural rigidity is fixed along the beam length.

Practically exact solutions of the local buckling problem for closed cross-section beams were obtained in papers [2, 3]. Later the same problem was solved in papers [4 – 6] in the case of simultaneous compression and bending.

The solutions obtained are very complex. At the same time, as GRAVES-SMITH [7] remarked, they do not enable the determination of load carrying capacity which is exhausted with the appearance of "crinkle modes", not considered in papers [7 – 9].

Intermediate stiffeners are widely used in many types of metal structures. These stiffeners carry a portion of the loads and divide the plate element into smaller sub-elements, thus increasing considerably the load-carrying capacity. The size and shape of intermediate stiffeners in thin-walled structures exerts a strong influence on the stability and post-buckling behaviour of the thin-walled structures.

The importance of the minimum rigidity of the intermediate stiffeners required to restrict buckling to the plate elements was studied, for example, in papers [10–14]. The test specimens, experimental works and comparisons made with design rules of plates and open cross-section structures were discussed in detail e.g. in [15, 16].

Before passing to the solution in the elasto-plastic range, one should determine the nonlinear solution in the elastic range which involves considering the transformation of local mode with the increase of load; this was done e.g. in [17–19].

A more comprehensive review of literature has been done e.g. in [19, 20].

In the present paper, the analysis of the interactive buckling of thin-walled beam-columns with intermediate stiffeners [13] is used, though regarding only the uncoupled buckling.

A semi-analytical method of solution of the bending problem for a thin-walled beam after local buckling is proposed and the beam flexural rigidity, without using the hypotheses on effective width of plates subject to bending, is determined based on the linear analysis of the local mode. The results obtained here are compared with the analytical and the experimental results obtained by other authors [15, 19]. This analysis, however, does not take into account the lateral buckling of beams.

2. Structural problem

The thin-walled prismatic beam of length l, composed of plane, rectangular plate segments interconnected along the longitudinal edges, simply supported at both ends and loaded by bending moment is considered (Fig. 1). A plate model is adopted for the beam (for a more detailed analysis see Appendix and [13]).

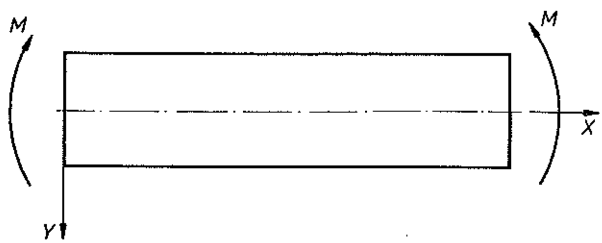

FIG. 1. Thin-walled beam.

The nonlinear problem is solved by asymptotic KOITER'S method [21]. Displacement $\overline{\mathrm{U}}$ and force $\overline{\mathrm{N}}$ fields are expanded into power series in the local buckling mode amplitude, ξ (divided by the thickness of the first component plate):

$$\begin{aligned}\overline{\mathbf{U}} &= \overline{\mathbf{U}}_i^{(0)} + \xi\overline{\mathbf{U}}_i^{(1)} + \xi^2\overline{\mathbf{U}}_i^{(2)} + \dots, \\ \overline{\mathbf{N}} &= \overline{\mathbf{N}}_i^{(0)} + \xi\overline{\mathbf{N}}_i^{(1)} + \xi^2\overline{\mathbf{N}}_i^{(2)} + \dots,\end{aligned} \tag{1}$$

where $\overline{\mathbf{U}}_i^{(0)}$, $\overline{\mathbf{N}}_i^{(0)}$ are the pre-buckling fields; $\overline{\mathbf{U}}_i^{(1)}$, $\overline{\mathbf{N}}_i^{(1)}$ – the buckling mode; $\overline{\mathbf{U}}_i^{(2)}$, $\overline{\mathbf{N}}_i^{(2)}$ the post-buckling fields.

The corresponding expression for the total potential energy for the local buckling mode of the perfect structure has the following form [19, 21, 22]:

$$\Pi = -a_0\lambda^2/2 + a_1\xi^2(1-\lambda/\lambda_{\mathrm{cr}})/2 + a_{111}\xi^3/3 + a_{1111}\xi^4/4, \tag{2}$$

where λ – load parameter, λ_{cr} – critical value of λ, $\Pi_0 = a_0\lambda^2/2 = M^2l/(2EI)$ – energy of pre-buckling bending, M – bending moment applied to the beam, I – the second moment of area of beam's cross-section. The coefficients a_1, a_{111}, a_{1111} are calculated by known formulas [21, 22].

The ratios of $\lambda/\lambda_{\mathrm{cr}}$ and M/M_{cr} (where M_{cr} is the critical value of bending moment for the local buckling mode) are used as equivalent parameters of loading. If $\lambda/\lambda_{\mathrm{cr}} = 1$ (i.e. $M/M_{\mathrm{cr}} = 1$) then $a_0 = M_{\mathrm{cr}}^2 l/(EI)$.

By substituting the expansion (1) into equations of equilibrium (A.2), junction conditions and boundary conditions, as well as boundary value problems of the zero, first and second order are obtained. The zero approximation describes the global pre-buckling beam bending. The first approximation, which is the linear problem of stability, is reduced to a system of homogeneous differential equations of equilibrium ([13]). This solution of the first order enables us to determine the local buckling mode and the critical value of bending moment, M_{cr} (performing a minimization with respect to the number of half-waves, m).

3. Overall bending after the local buckling

Global flexural stiffness is determined by the angle of beam edge rotation which equals $\vartheta = \partial\Pi/\partial M$ ([19]). Equation (2) gives

$$\vartheta = -a_0M/M_{\mathrm{cr}}^2 - a_1\xi^2/(2M_{\mathrm{cr}}). \tag{3}$$

The angle of rotation up to the local buckling is

$$\vartheta_0 = -a_0M/M_{\mathrm{cr}}^2. \tag{4}$$

The ratio ϑ_0/ϑ of the angles of rotation defines the coefficient of the reduced flexural rigidity, $\eta = I_*/I$, where I_* is the second effective moment of area of the cross-section:

$$\eta = \left[1 + \frac{a_1}{a_0}\frac{M_{cr}}{2M}\xi^2\right]^{-1}. \tag{5}$$

As in the papers [7, 19], the state of deformation is defined by the curvature ratio, $\chi_0 = (1/R)/(1/R_{cr})$, where R_{cr} is the radius of curvature for $M = M_{cr}$ which can be expressed as follows:

$$\chi_0 = (M/I_*)/(M_{cr}/I) = M/M_{cr}(1/\eta). \tag{6}$$

A "lower bound" approach has been used by KOITER and PIGNATARO [1] (see also [23]) to obtain a post-buckling stiffness coefficient η for a general local buckling mode described by

$$w_i^{(1)}(x,y) = W_i^{(1)}(y)\sin\frac{m\pi x}{l}, \tag{7}$$

where m is the number of axial half-waves of the local buckling mode and the transverse wave profiles are defined by the functions $W_i^{(1)}(y)$ for the i-th wall of the beam. The "lower bound" value of η is given by

$$\eta = 1 - \left(\overline{W}^2\right)/\overline{W}^4, \tag{8}$$

where $\overline{W}^2$ and $\overline{W}^4$ are the average values of $[W_i^{(1)}(y)]$ and $[W_i^{(1)}(y)]$, respectively, over the cross-section,

$$\begin{aligned}\overline{W}^2 &= \sum_i \frac{1}{b_i}\int_0^{b_i}\left[W_i^{(1)}(y)\right]^2 dy_i,\\ \overline{W}^4 &= \sum_i \frac{1}{b_i}\int_0^{b_i}\left[W_i^{(1)}(y)\right]^4 dy_i.\end{aligned} \tag{9}$$

The functions defining the first order displacement are determined by the method of transition matrices in the same way as in [13].

In paper [1] KOITER and PIGNATARO have shown that for uniform compressed panels, the value of η obtained from Eq. (8) is a "lower bound" with respect to the exact value. In this case a constant value of η is obtained. It is not difficult to find out from (6) that in the adopted system of coordinates, $M/M_{cr} - \chi_0$, the post-buckling elastic path of equilibrium is a straight line.

Just such a path was obtained in [19] by taking into account the change of η as a function of the load parameter λ.

A "lower bound" approach by Koiter and Pignataro is applied here for structures subjected to eccentric compression and bending.

Figure 2 presents the dependences of the post-buckling stiffness coefficients on the load parameter, λ/λ_{cr}, defined by Eqs. (5) (as it was done in [19]) and (8) for a square box-beam (analyzed in [19]) subject to bending, having the following dimensions:

$$b = 100 \text{ [mm]}, \qquad t = 1.25 \text{ [mm]}, \qquad l = 700 \text{ [mm]}.$$

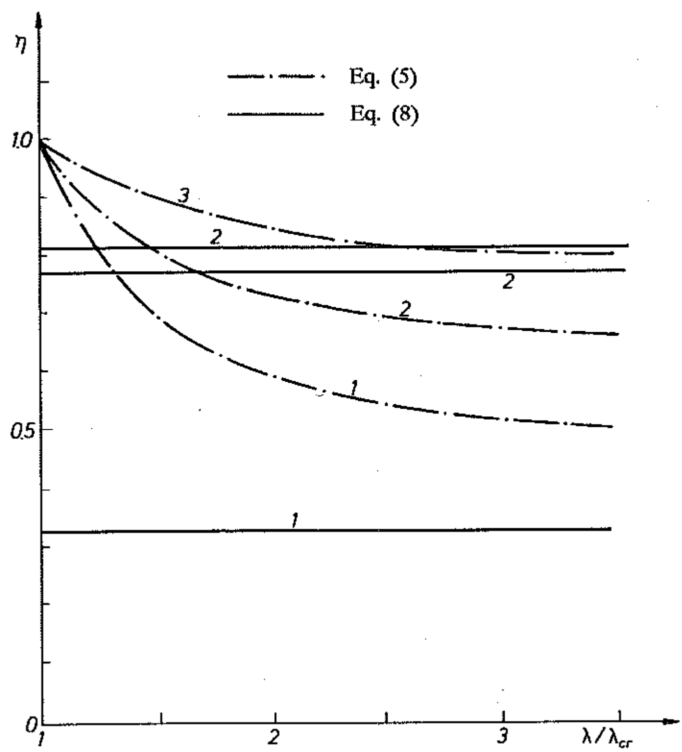

FIG. 2. Post-buckling stiffness coefficient, η, as a function of load parameter, λ/λ_{cr} defined by equations (5) and (8) for bending of the square box-beam. Curves: *1* – uniform compression, *2* – eccentric compression, *3* – bending.

This figure shows also the above relationships obtained on the basis of papers [17–19] and of the equation (8) (like in [13, 14]) for the uniform compression and for the triangular distribution of external load along the webs. Papers [17–19] take into consideration the influence of transformation of the local buckling mode upon the values of η.

For $\lambda/\lambda_{cr} \geq 1$, the coefficient η decreases abruptly from $\eta = 1$ to a value given by (5) and in a step-like manner for (8).

In the case of eccentric compression and bending, the lines representing relationships (5) and (8) intersect each other. In these cases the post-buckling stiffness coefficients η given by (8) cannot be always called the "lower bound" value of the η, as it was done in [1, 23] (compare also curves 2 and 5 in [17], Fig. 4).

Average dimensions of the beam specimens analyzed in [15] are given in Table 1. The geometry of cross-sections is shown in Fig. 3.

Table 1. Dimensions of channel beams (in mm).

Spec.no.	l	w	b_s	b_{sp}	b_d	b_w	b_l	t	w/t
LCO	1060	76.95	0.0	0.0	0.0	78.5	15.0	1.570	49.0
*A*0	1060	76.60	0.0	0.0	0.0	77.1	15.1	0.410	186.8
*A*1	1060	71.50	5.2	11.2	4.2	76.4	17.3	0.405	176.8
*A*2	1060	73.10	6.5	8.1	5.55	77.4	17.5	0.410	178.2
*A*3	1060	73.55	6.6	7.1	9.0	76.8	16.6	0.410	179.4
*A*4	1060	73.65	6.8	7.2	11.8	76.9	18.0	0.415	177.4
*A*5	1060	73.35	6.6	8.0	18.2	76.9	17.1	0.405	181.2
*B*1	1060	70.55	4.7	13.2	6.0	76.9	17.6	0.682	103.5
*B*2	1060	72.70	6.5	8.8	5.6	76.7	17.4	0.691	105.2
*B*3	1060	73.10	6.5	7.8	7.7	77.1	18.5	0.698	104.7
*B*4	1060	73.20	6.7	8.0	10.7	77.0	18.9	0.695	105.3
*C*0	1060	76.90	0.0	0.0	0.0	77.4	15.3	0.818	94.0
*C*1	1060	70.75	4.5	12.7	5.65	77.0	17.2	0.818	86.4
*C*2	1060	72.65	5.7	9.5	5.45	76.7	17.7	0.821	88.5
*C*3	1060	73.85	6.9	7.3	8.7	77.4	17.1	0.814	90.7
*C*4	1060	73.60	7.0	7.4	11.7	77.0	18.5	0.815	90.3
*C*5	1060	73.35	6.9	7.6	18.2	77.5	18.2	0.810	90.6
*D*1	1060	71.25	5.5	9.5	4.55	76.5	16.2	0.759	93.9
*D*2	1060	72.50	6.5	8.0	8.95	76.4	25.5	0.761	95.2
*D*3	1060	72.15	6.2	8.2	11.35	76.2	16.8	0.762	94.6
*D*4	1060	72.40	6.2	8.2	15.1	76.9	9.0	0.763	94.8
*E*1	1060	71.65	7.0	11.0	4.85	76.3	17.0	1.213	59.1
*E*2	1060	72.30	7.0	10.0	5.1	77.8	17.0	1.207	59.9
*E*3	1060	73.55	7.0	7.0	8.2	79.0	18.0	1.207	60.9
*E*4	1060	73.20	7.0	8.1	9.0	77.2	14.5	1.200	61.0
*E*5	1060	73.80	7.7	7.7	12.0	76.7	17.0	1.209	61.0
*E*6	1060	73.80	7.5	7.5	18.8	78.7	17.5	1.202	61.2
*F*1	1060	70.25	5.2	13.0	5.1	76.0	17.7	1.519	46.2
*F*2	1060	70.90	5.5	12.0	6.0	75.5	17.8	1.528	46.4
*F*3	1060	71.75	6.2	10.0	9.7	76.3	17.0	1.528	46.9
*F*4	1060	71.50	6.5	10.5	12.55	76.3	15.5	1.514	47.2
*F*5	1060	71.65	7.0	9.75	18.25	77.6	18.5	1.525	47.0

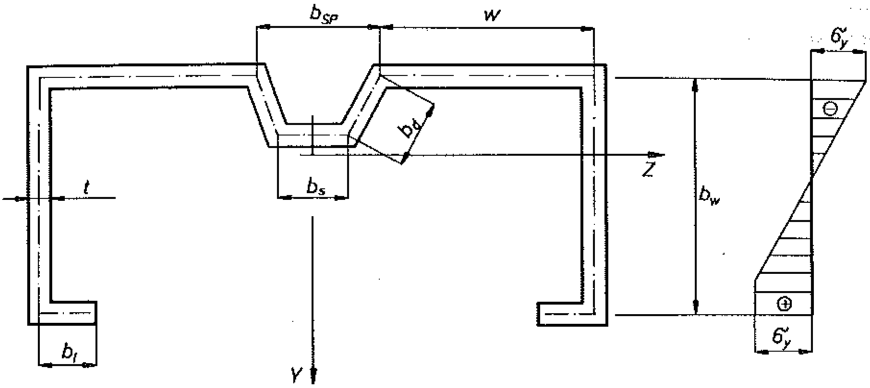

FIG. 3. The cross-section of the beam specimens.

The value of Young's modulus of elasticity and Poisson's ratio are assumed, respectively, $E = 205$ GPa and $\nu = 0.3$.

The average values of the yield strength σ_y for each material thickness are given in Table 2.

Table 2. Average yield strengths.

Test series	σ_y [MPa]
A	275.0
B	374.0
C	285.0
D	147.4
E	176.5
F	214.5

In Table 3 the lowest values of critical bending moments, M_{cr}, are listed, referring to the local buckling; the table contains also the number of half-waves, m, experimental values of critical moments ([15]), $\overline{M}_{cr}$, abbreviated designations of local buckling mode as well as the collapse data.

In [15] the experimentally obtained values of critical loads are given only for 13 out of the 32 cases studied.

The beam reinforced with intermediate stiffeners show a different local buckling mode. In the papers [14, 16] local buckling modes presented in Fig. 4 are denoted as follows: local distortional mode – LDM (Fig. 4a), local symmetric mode – LSM (Fig. 4b) and local antisymmetric mode – LAM (Fig. 4c). The values of the critical stress corresponding to the local symmetric mode and the local antisymmetric one differ insignificantly or are nearly identical (for a more detailed analysis see [13, 14]).

Table 3. Collapse data of channel beams.

Spec. no.	M_{cr}	m	$\overline{M}_{cr}$	κ	η	M_*	$\overline{M}_E$	$(M_* - \overline{M}_E)/M_*$	buckling mode
LCO	1986.5	9		0.6617	0.8697	2257.0	3223.1	−0.428	LSM
*A*0	35.0	9		0.6415	0.8605	575.1	339.0	0.696	LSM
*A*1	161.2	19	120.1	0.0965	0.5514	486.0	350.8	0.278	LAM
*A*2	170.2	19		0.6085	0.8608	612.5	375.7	0.386	LAM
*A*3	167.8	19	149.6	0.5963	0.8606	591.4	349.6	0.409	LAM
*A*4	175.9	19		0.5908	0.8602	619.6	318.9	0.485	LAM
*A*5	161.5	19	164.4	0.5782	0.8589	587.6	306.4	0.478	LSM
*B*1	699.4	4	605.2	0.5819	0.8488	1126.9	1177.8	−0.045	LDM
*B*2	801.9	19	634.8	0.3255	0.7214	1286.8	1250.0	0.028	LAM
*B*3	837.8	19	605.2	0.6008	0.8598	1443.0	1291.4	0.105	LAM
*B*4	831.8	19	581.0	0.5928	0.8595	1444.9	1273.3	0.118	LAM
*C*0	277.8	9	226.6	0.6624	0.8699	1202.8	1232.7	−0.024	LDM
*C*1	931.8	4	818.2	0.0866	0.5493	1021.8	1395.0	−0.365	LDM
*C*2	1229.7	3	1125.8	1.0	1.0	1229.7	1489.6	−0.295	LDM
*C*3	1308.5	19	1220.4	1.0	1.0	1308.5	1463.0	−0.173	LAM
*C*4	1328.9	19	1037.0	1.0	1.0	1328.9	1380.2	−0.079	LAM
*C*5	1311.3	19	1149.5	1.0	1.0	1311.3	1377.2	−0.088	LAM
*D*1	840.8	4		1.0	1.0	840.8	889.2	−0.057	LDM
*D*2	1127.6	19		1.0	1.0	1044.7	1023.4	0.020	LAM
*D*3	1084.3	19		1.0	1.0	913.5	924.2	−0.011	LAM
*D*4	1027.3	19		1.0	1.0	810.8	877.3	−0.082	LAM
*E*1	2580.2	4		1.0	1.0	1755.6	1909.0	−0.087	LDM
*E*2	2800.9	4		1.0	1.0	1802.9	1904.9	−0.056	LDM
*E*3	4352.2	18		1.0	1.0	1877.5	2045.1	−0.089	LAM
*E*4	4113.0	19		1.0	1.0	1698.3	1930.8	−0.136	LAM
*E*5	4245.2	18		1.0	1.0	1765.7	1980.8	−0.121	LAM
*E*6	4314.8	19		1.0	1.0	1836.7	1980.8	−0.078	LAM
*F*1	3446.8	6		1.0	1.0	2673.4	2717.1	−0.016	LDM
*F*2	4581.2	5		1.0	1.0	2671.6	2918.3	−0.092	LDM
*F*3	8238.2	3		1.0	1.0	2679.7	2924.2	−0.091	LDM
*F*4	8420.4	19		1.0	1.0	2585.2	2687.6	−0.039	LAM
*F*5	8956.5	19		1.0	1.0	2797.8	2829.5	−0.011	LAM

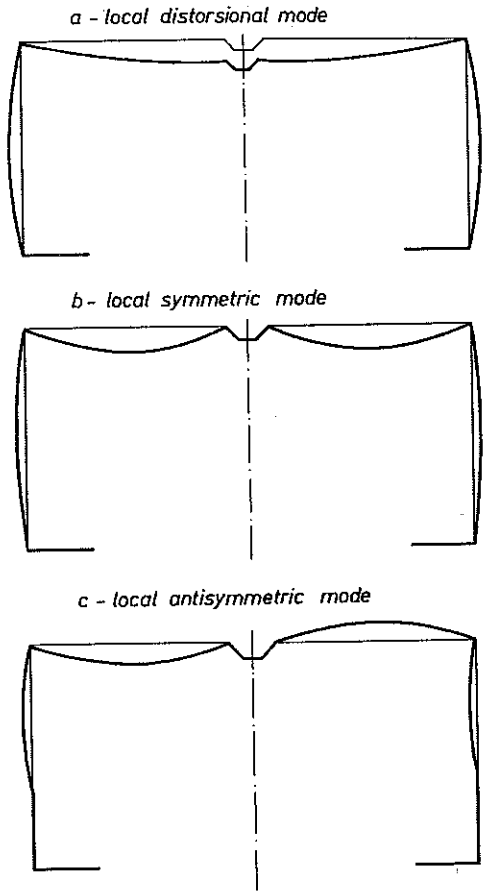

FIG. 4. Local buckling modes for the beam with intermediate stiffeners.

In the analyzed cases, experimental values of critical bending moments are lower than the theoretical ones; this can be explained by a coupled buckling of local buckling modes from the very start of the loading process ([13, 14]).

4. Determination of load carrying capacity

For a post-buckling analysis in the elastic range, it is only possible to obtain an approximate estimation of load carrying capacity on the basis of a simplified threshold criterion.

In this paper, the following criterion is adopted for the load carrying capacity, M_*:

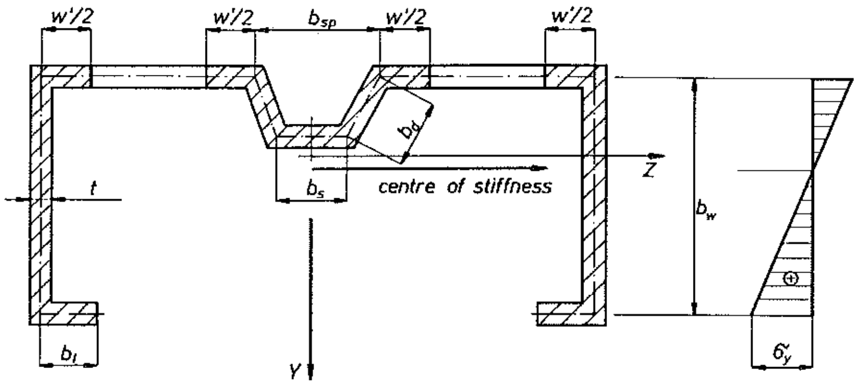

FIG. 5. Effective cross-section of transverse beam and stress distributions for the adopted criterion of the load carrying capacity.

• in a plate under tension, the yield stress is attained at a limit-load value higher than the critical moment, M_{cr}, that is at $\eta < 1$ (Fig. 5). In a compressed plate, elastic strains are present;

• in a compressed plate, the yield stress is attained at a limit-load value lower than the critical moment, M_{cr} (Fig. 3). In this case we are dealing with pre-buckling bending, hence it is assumed that $\eta = 1$;

• limit-moment value is equal to the critical moment, M_{cr}, when the value of stresses in a stretched plate referring to M_{cr}, determined for pre-buckling stiffness (i.e. $\eta = 1$), is lower than the yield stress, and the value of stresses in the same plate, referring to M_{cr} and determined for reduced flexural stiffness (i.e. $\eta < 1$) is higher than the yield stress.

Such a criterion takes into account the post-buckling of plates under compression, or a lack of this buckling in the pre-buckling state of the perfect structures; it considers also a relevant mechanism of failure by yielding of the plate being compressed.

However, in order to determine maximum stresses in the plate after local buckling of the beam, one must find not only the reduced flexural stiffness, but also the position of effective stiffness center of the cross-section.

As a calculation examples, the above thin-walled beams are considered. Generally speaking, one has to find the effective width of a plate under compression and of webs subject to bending. In this paper only the width of a compressed flange is reduced (Fig. 5) to obtain the real decrease in flexural stiffness of the cross-section after local buckling.

Quantity $\kappa = w'/w \geq 0$, where w' is the effective width of the compressed plate, is derived from the condition that the second moment of area of effective cross-section, I_*, should corresponds to the expression for the coefficient of reduced flexural stiffness, η (8) (that is $I_* = \eta I$). Expressing the deviation of the centroid of the effective cross-section via κ and determining the second moment of area of the effective cross-section with respect to the neutral axis displaced by e the relationship between κ and η is obtained (analogous with in [19]).

It is obvious that the applied method of reducing the cross-section gives somewhat too high values of e and of stresses. Therefore this approach provides a lower bound of the load carrying capacity.

With reference to the beams discussed above, Table 3 contains also the coefficients κ and η, the theoretical and the experimental limit load values (M_* and $\overline{M}_E$, respectively) and their relative differences. The values of $\overline{M}_E$ are taken from [15].

In all the analyzed cases, when $w/t \leq 105$, a good agreement is found between the presented lower estimate of the load carrying capacity and the experimental results (cases B, C, D, E, F). Only in $B3$ and $B4$ the theoretical values of the limit load, M_* are by about 10% higher than $\overline{M}_E$ obtained experimentally.

In cases belonging to series A (that is $w/t = 180$), the calculated limit load values are significantly higher than those obtained experimentally.

In [24] the following values of local imperfections, f_0, were adopted on the basis of experimental data:

$$f_0/t = 0.00004(b/t)^2 \qquad \text{for} \quad b/t \leq 150,$$
$$f_0/t = -0.9 + 0.012(b/t) \qquad \text{for} \quad b/t \geq 150.$$

In series A the magnitude of the local imperfections is $f_0/t = 1.26$ (for $w/t = 180$), while in $B - f_0/t = 0.441$ ($w/t = 105$). In [19] a very significant influence is shown of local imperfections upon the post-buckling limit load of beams subjected to bending.

Large discrepancies between the theoretical and experimental results in series A can, therefore, be explained by such factors as the influence of imperfections, low flexural rigidity, residual stresses, and the fact that the determined value of η coefficient is not its lower estimate (see notes to Fig. 2). In this case the assumption of lower estimate of the load carrying capacity is not fulfilled.

The analysis presented here provides a correct evaluation of the load carrying capacity for $b/t \leq 120$ and for moderate imperfections.

If, as it was done in [19], it was considered that local imperfections had an effect upon η, and, consequently, upon the load carrying capacity, the latter could be properly evaluated also in case of $b/t \geq 120$ and of greater local imperfections (see Fig. 2 in [19]).

5. Conclusions

The present paper deals with a "lower bound" approach by Koiter and Pignataro which is applied for determining the post-buckling flexural stiffness of the elastic beam with a stiffener. The overall bending of the beam is also included in the analysis. Plate elements are adopted for modelling the beam structure. An approximate evaluation of the load carrying capacity is presented.

Appendix

The thin-walled beam with central intermediate stiffeners simply supported at the ends and loaded by a bending moment is considered. For each plate component accurate geometrical relationships are assumed to take into account both out-of-plane and in-plane bending ([13]):

$$
\begin{aligned}
\varepsilon_{ix} &= u_{i,x} + 0.5(u_{i,x}^2 + v_{i,x}^2 + w_{i,x}^2),\\
\varepsilon_{iy} &= v_{i,y} + 0.5(u_{i,y}^2 + v_{i,y}^2 + w_{i,y}^2),\\
2\varepsilon_{ixy} &= \gamma_{ixy} = u_{i,y} + v_{i,x} + u_{i,x}u_{i,y} + v_{i,x}v_{i,y} + w_{i,x}w_{i,y}\,,
\end{aligned}
\tag{A.1}
$$

$$
\kappa_{ix} = -w_{i,xx}\,, \qquad \kappa_{iy} = -w_{i,yy}\,, \qquad \kappa_{ixy} = -w_{i,xy}\,.
$$

where i is the plate number, $(..)_{,x} = \partial(..)/\partial x$, $(..)_{,y} = \partial(..)/\partial y$.

The differential equilibrium equations resulting from the virtual work principle and corresponding to the expressions (A.1) for the i-th wall can be written as follows:

$$
\begin{aligned}
&N_{ix,x} + N_{ixy,y} + (N_{ix}u_{i,x})_{,x} + (N_{iy}u_{i,y})_{,y}\\
&\qquad\qquad + (N_{ixy}u_{i,x})_{,y} + (N_{ixy}u_{i,y})_{,x} = 0,\\
&N_{iy,y} + N_{ixy,x} + (N_{ix}v_{i,x})_{,x} + (N_{iy}v_{i,y})_{,y}\\
&\qquad\qquad + (N_{ixy}v_{i,x})_{,y} + (N_{ixy}v_{i,y})_{,x} = 0,\\
&D_i\nabla\nabla w_i - (N_{ix}w_{i,x})_{,x} - (N_{iy}w_{i,y})_{,y}\\
&\qquad\qquad - (N_{ixy}w_{i,x})_{,y} - (N_{ixy}w_{i,y})_{,x} = 0.
\end{aligned}
\tag{A.2}
$$

The solution of these equations for each plate should satisfy the kinematic and static conditions at the junctions of adjacent plates and the boundary conditions at the ends $x = 0$ and $x = l$.

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