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A STUDY OF THE BRAZILIAN TEST

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The first part of the present paper is devoted to the analysis of the standard Brazilian test, based on the theory of elasticity. Some literature data on the differences between the tensile strength S_r in the usual sense and the approximate strength S_{rb} are quoted for some types of French rocks. The results of the present author's experimental studies of three types of Polish rocks are discussed, the causes of the differences between S_r and S_{rb} being analysed. In the second part of the paper it is proposed to consider the Brazilian test as a problem of the theory of limit states, showing that the assumption of a modified Coulomb condition is in agreement with the mode of the fracture of the test specimen used. It is found that such a method leads to quite realistic approximate values of the tensile strength.

1. Introduction

The Brazilian test of brittle materials which consists in transverse compression of test specimens of various forms makes it possible to obtain approximate values of the tensile strength of such a material. Despite the fact that many works concerned with the Brazilian test have already been published, this method of testing still remains an object of studies aimed at its interpretation. The present paper, which is a continuation of the considerations of [1], presents the results of direct and indirect tensile tests of some Polish rocks by applying the Brazilian method, which is analysed on the grounds of the theory of limit states.

1.1. Description of the test

The use of indirect, approximate methods is a consequence of the difficulties which are met with if direct, ordinary tensile test is applied to brittle materials [2]. Some of those methods are illustrated in Fig. 1, the most popular being those of transverse compression of thin discs with a load p distributed over a small area (Fig. 1 a). This is the ordinary, standard Brazilian test. Other tests consist in compression of discs with a small central hole, of plate strips or other types of specimens [3].

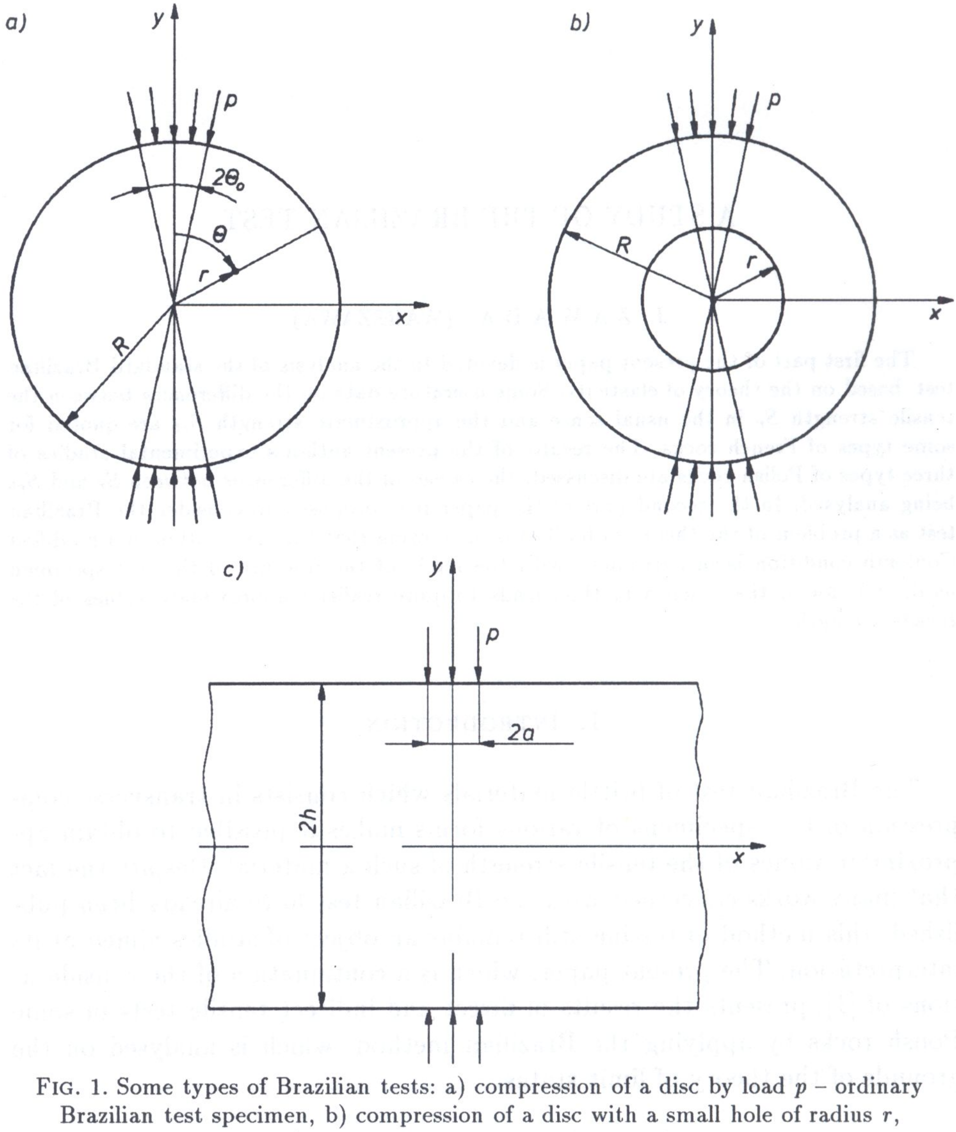

FIG. 1. Some types of Brazilian tests: a) compression of a disc by load p – ordinary Brazilian test specimen, b) compression of a disc with a small hole of radius r, c) compression of a strip of material.

1.2. Theory of the standard Brazilian test (Fig. 1 a)

This theory is based on the assumption that the material of the disc is perfectly elastic. If a disc of radius R and thickness t is loaded by a pressure p acting over areas bounded by angles $2\theta_0$, methods of the theory of elasticity can be used to determine the state of stress at any point of the disc. The principal stresses σ_1 and σ_2 varying along the y-axis (compressive stress being assumed to be positive) are expressed by the formulae (Fig. 1 a).

$$(1.1)\qquad \begin{aligned} \sigma_1 &= \frac{2p}{\pi}\left[\frac{(1-k^2)\sin 2\theta_0}{1-2k^2\cos 2\theta_0+k^4}+\operatorname{arc\,tg}\left(\frac{1+k^2}{1-k^2}\operatorname{tg}\theta_0\right)\right],\\ \sigma_2 &= -\frac{2p}{\pi}\left[\frac{(1-k^2)\sin 2\theta_0}{1-k^2\cos 2\theta_0+k^4}-\operatorname{arc\,tg}\left(\frac{1+k^2}{1-k^2}\operatorname{tg}\theta_0\right)\right],\end{aligned}$$

where $k = r/R$.

In the particular case of $R = 25$ mm, $2\theta_0 = 4°$, $p = 19.5$ MPa the variation of those stresses is as represented in Fig. 2 a. The stress σ_1 is compressive for any value of r. The stress σ_2 is tensile and approximately uniform over a considerable part of the diameter and becomes compressive in the neighbourhood of the region of contact. If the loaded area decreases, the stress σ_2 tends to a constant value

$$(1.2)\qquad \lim_{\theta_0\to 0}\sigma_2 = -\frac{P}{\pi Rt}, \qquad 0 \le r < R,$$

where P is the loading force: $P = 2tR\sin\theta_0 p \approx 2tR\theta_0 p$.

In the case of a concentrated load, that is if $2\theta_0 \to 0$, the stress σ_2 is tensile over the entire diameter. The expression (1.2) determines an approximate tensile strength, which will be denoted S_{rb} for distinction from the uniaxial tensile strength S_r.

To complete this information let us observe that the theory of the Brazilian test illustrated in Fig. 1 b has been developed by D.W. HOBBS [4], and the theory of the test shown in Fig. 1 c (for concentrated loads only ($2a \to 0$)) – by L.N. Filon, whose solution is mentioned in the monograph [5].

1.3. Comparison of the uniaxial tensile strength S_r with the approximate strength S_{rb}

Papers comparing the values of strength obtained by the method of uniaxial tension and the Brazilian method are seldom met with in the literature, the opinion that there are some discrepancies between them being dominant, however. From the tests performed by A. JULIEN [6] with different rock types of French origin it follows that the strength S_{rb} is higher than S_r (Table 1).

The differences between S_r and S_{rb} are considerable. The quantity S_{rb} may attain values of up to three times that of S_r.

Below (Sec. 3) we shall analyse the causes of those discrepancies, but first, in order to facilitate further analysis let us describe our own experimental investigation and discuss the results obtained.

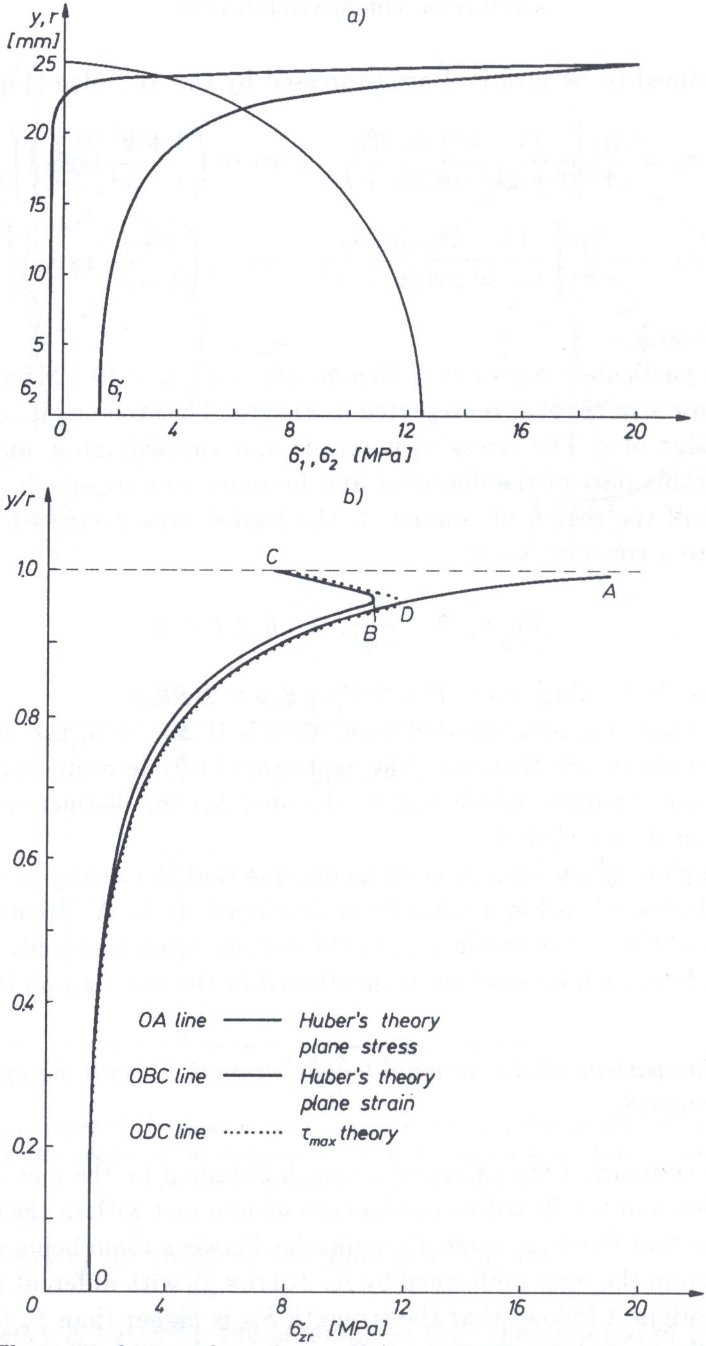

FIG. 2. The state of stress along the y-axis ($p = 19.5$ MPa, $R = 25$ mm, $2\theta_0 = 4°$): a) diagram of principal stresses σ_1 and σ_2, b) reduced stress according to the Huber theory and the theory of maximum shear stress in the case of plane of stress (Poisson's ratio $\nu = 0.3$).

Table 1. Direct tensile strength S_r and the approximate strength S_{rb} as obtained by the method of the Brazilian test for various rocks (some of the data published by A. JULIEN [6]).

No	Type of rock	S_r [MPa]	S_{rb} [MPa]	$n_1 = S_{rb}/S_r$
1	limestone (Villette)	27	80	2.96
2	limestone (Misset)	40	85	2.12
3	limestone (Marquise)	60	120	2.00
4	limestone (Hauteville)	60	100	1.66
5	granite (Gérardmer)	80	145	1.81
6	granite (Flamanville)	70	137	1.95
7	granite (Cap de Long)	38	112	2.94
8	granite (la Bresse)	60	100	1.66
9	basalt (Naon l'Etape)	330	360	1.09

2. Experimental study

Direct (ordinary) and Brazilian indirect tensile tests were performed, the latter consisting in transverse compression of cylindrical specimens by means of narrow, flat and wedge-shaped punches (Fig. 1 a). Three types of Polish rocks were tested, namely soft limestone of the Pińczów region (Pińczów limestone), medium-grain marble of the region of Stronie ("Biała Marianna" marble), and compact limestone of the Kielce region ("Morawica" limestone). Those rocks had been the objects of earlier extensive experiments concerned with their properties in simple and general states of stress, limit states, crushing energy etc. Some of the results obtained are discussed in the monograph [7].

2.1. Experimental technique

The test specimens used had been cut out from rectangular plates, previously prepared for the purpose, by means of diamond bits with an inner radius of $2R = 24.7$ mm or 14.8 mm. The lateral surfaces of the specimens used for ordinary tensile tests were subjected to additional polishing operation. Next, the end parts of those specimens were joined by means of a two-component glue (an epoxy resin and a hardener [1]) with bushes of

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[1] The epoxy resin mentioned is known as epichlorohydrin and the hardener – as dimethylorminopropylamin, the trade names being Araldit AW 21101 and Hörter HW 2951, respectively. The two components were mixed in the weight proportion of 1:1. The gelation process lasted 5 min and full strength of the joint was reached in 48 hours.

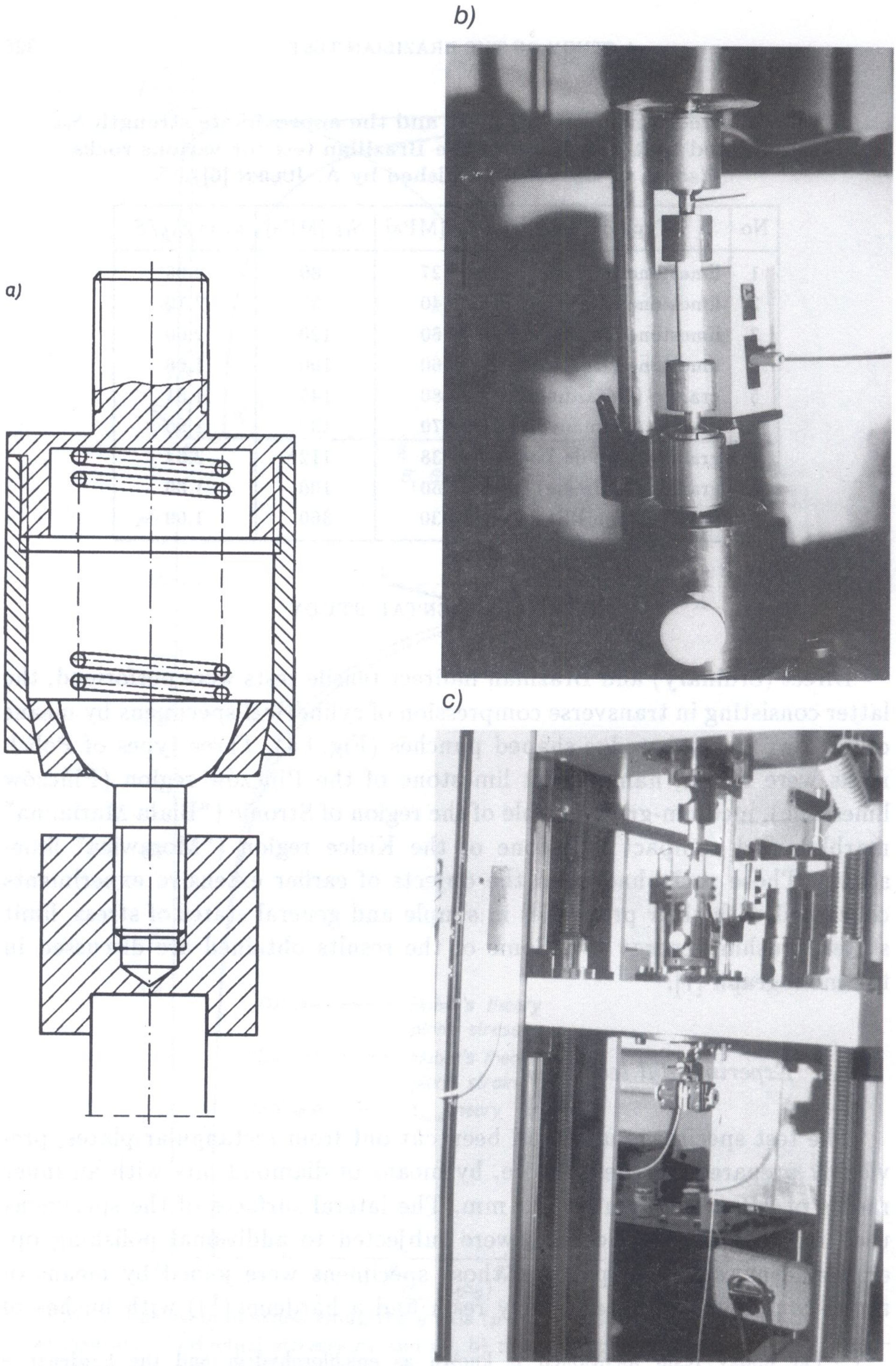

FIG. 3. A device for ordinary tensile test: a) a holder with a bush (sectional view), b) view of the holders with the test specimen, c) view of the test stand.

appropriate form. These bushes were fastened to special holders preventing action of bending moments and mounted on a ZWICK 1484 testing machine. Figure 3 shows the structural elements of a holder, a view of the assembly including the test specimen, the bushes and the holders and the complete test stand.

The test specimens were compressed by means of narrow, flat or wedge-shaped punches, using a device ensuring good alignment of the load with the tools. The loading rate was low and reached the value $v_0 = 1.5\,\mathrm{mm/min}$ for direct tension and $v_0 = 5\,\mathrm{mm/min}$ in the case of the Brazilian method, the loads thus being quasi-static.

2.2. Results of the experiment

The numerical results obtained from the direct tensile test and the Brazilian method are specified in Table 2, which includes also some information on the influence of the length of a compact limestone (limestone M) specimen on the state of stress and strain which may approach, under small values of the parameter t, a state of plane stress and, for higher values of t, a state of plane strain. Figure 4 shows some examples of mechanical characteristics for the ordinary tensile test and compression by punches (Brazilian test). Figure 5 illustrates some typical forms of fracture observed in the experiments.

2.3. Comparison of the strength S_{rb} with S_r

The experiments performed showed some discrepancies between the strength S_r determined by ordinary tensile test and the value of S_{rb} obtained by means of the Brazilian test. Compression by wedges gives a value of the strength which is lower than S_r and the use of punches (standard test) raises that value. The strength S_{rb} is distinctly influenced by the length (thickness) t of the specimen. A decrease in t results in an increase in S_{rb}. An increase in t makes S_{rb} tend to a constant value (Fig. 6), for limestone M compressed with punches, which means a scale effect. The low decrease in S_{rb} observed in Fig. 6 for the greatest two specimen lengths suggests that there were some inaccuracies in performing the experiments, that is the contact between the punches and the specimen was incomplete. Let us assume that the strength S_{rb} is equal to the results obtained in the case in which the diameter of the specimen is equal to its length t (Table 2, items 12, 14 and 16) and compare them with the results of ordinary tensile tests (Table 3).

Table 2. Numerical data on the specimens, loads and strengths S_r and S_{rb}.

1	2	3	4	5	6	7		8	9
No	Type of rock	$2R$ [mm]	t [mm]	n	$\overline{P}$ [kN]	$\overline{S}_r$	$\overline{S}_{rb}$ [MPa]	$\overline{P}_j$ [kN/mm]	Remarks
Ordinary tensile test									
1	soft limestone P	24.7	60	8	0.85	1.78	0.45 0.25		
2	medium marble BM	14.8	60	8	0.83	4.84	1.7 0.68		
3	compact limestone M	14.8	40	8	1.23	7.15	0.64 1.11		
Brazilian test (Compression by wedges $2\gamma = 30°$)									
4	Limestone P	24.7	8.0	6	0.23	0.76	0.22 0.17		
5	Marble BM	24.7	8.0	6		3.24	1.01 0.69		
6	Limestone M	24.7	8.0	6	1.55	5.02	0.92 0.80	0.195	*
7			12,0	6	2.04	4.38	1.13 0.77	0.170	*
8			16.0	6	2.69	4.34	1.45 0.96	0.168	*
9			20.0	6	2.76	3.57	0.94 0.68	0.139	*
10			24.0	6	3.83	4.12	0.93 0.67	0.158	*
Brazilian test Compression by narrow punches $2a = 3\,\mathrm{mm}$ ($2a = 2\,\mathrm{mm}$ measurements 17–25)									
11	Limestone P	24.7	8.0	6	0.72	2.24	0.36 0.25		
12		24.7	25.0	5	1.96	1.93	0.43 0.15		
13	Marble BM	24.7	8.0	6	3.28	10.26	3.54 4.20		
14			25.0	5	7.58	7.84	0.81 1.43		

* Investigation into the influence of length t of the test specimen on the realization of a plane state of strain.

Table 2 [cont.]

Ordinary tensile test								
1	2	3	4	5	6	7	8	9
No	Type of rock	$2R$ [mm]	t [mm]	n	$\overline{P}$ [kN]	$\overline{S}_r$ $\overline{S}_{rb}$ [MPa]	$\overline{P}_j$ [kN/mm]	Remarks
15	Limestone M	24.7	8,0	6	5.38	$16.74^{2.75}_{2.63}$		**
16			25.0	5	11.41	$11.93^{1.97}_{1.13}$		**
17		14.8	8.0	6	3.52	$17.77^{4.76}_{5.82}$		**
18			16.0	6	5.51	$13.88^{3.53}_{2.95}$		**
19			24.0	6	7.82	$13.14^{3.93}_{4.87}$		**
20			32.0	6	9.96	$12.56^{4.10}_{.86}$		**
21			40.0	6	12.36	$12.47^{4.12}_{2.57}$		**
22			48.0	6	16.24	$12.40^{3.10}_{3.47}$		**
23			56.0	6	16.96	$12.22^{3.52}_{3.32}$		**
24			64.0	6	18.43	$11.62^{3.70}_{3.02}$		**
25			70.0	6	20.08	$11.53^{3.31}_{2.17}$		**

** Investigation into the influence of the length t of the test specimen on the realization of a plane state of strain.

Symbols used

$2R$ – diameter of the specimen, t – thickness (length) of the specimen, n – number of specimens in a series, $\overline{P}$ – mean value of the limit (cracking) force in a series, $\overline{S}_r$ – mean value of uniaxial tensile strength as obtained by the direct method, $\overline{S}_{rb}$ – uniaxial tensile strength as obtained by the Brazilian method, $\overline{P}_j$ – mean unit force, that is force per unit length of the specimen, 2γ – apex angle of the wedges, $2a$ – width of the punch.

The indices used with S_r and S_{rb} in column 7 indicate the upper and lower deviation of the maximum and minimum value of the strength from the mean value for a series.

The values of the limit forces assumed for analysis were those of the maximum forces attained in particular tests. The strength S_r was evaluated from the formula $\overline{S}_r = \overline{P}/\Pi R^2$ and $\overline{S}_{rb}$ – from relation $(1.1)_1$ for the centre of symmetry, $k = 0$.

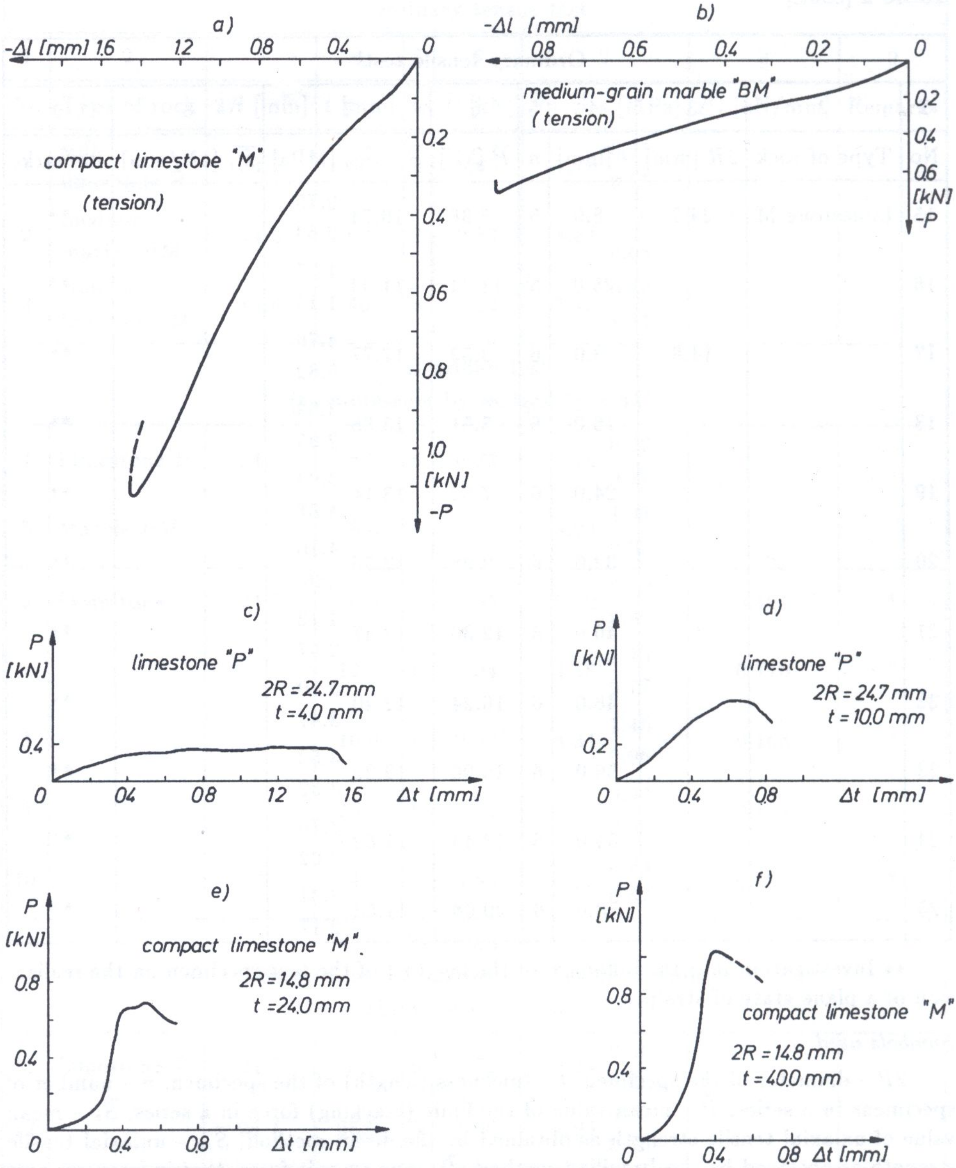

FIG. 4. Example of mechanical characteristics for direct tension and compression by the Brazilian method: a, b) direct tension of limestone M and marble BM; c, d, e, f) compression of specimens of limestone P and M.

The differences are the highest for the medium-grain marble and the compact limestone, the lowest being those obtained for soft limestone. The above results approach some of the data obtained for French rocks by A. Julien (Table 1).

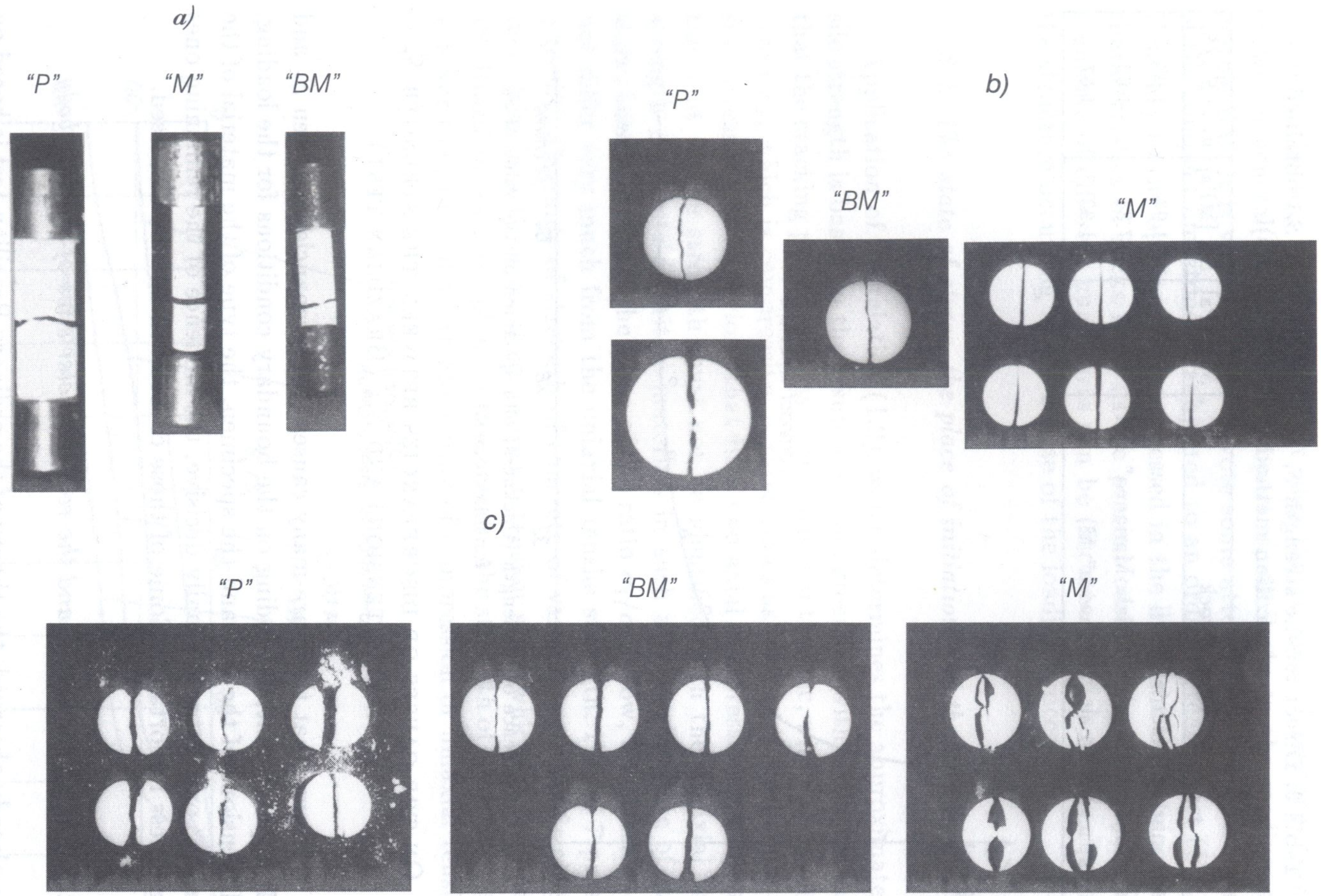

FIG. 5. Examples of fracture modes: a) direct tension, b) Brazilian method – compression by wedges, c) Brazilian method – compression by punches.

Table 3. Direct tensile strength S_r and the strength S_{rb} obtained by the Brazilian method (author's results).

No	Type of rock	S_r [MPa]	S_{rb} [MPa]	$n_1 = S_{rb}/S_r$
1	soft marble (Pińczów P)	1.78	1.93	1.08
2	medium-grain marble ("Biała Marianna" BM)	4.84	7.84	1.62
3	compact limestone ("Morawica" M)	7.15	11.93	1.67

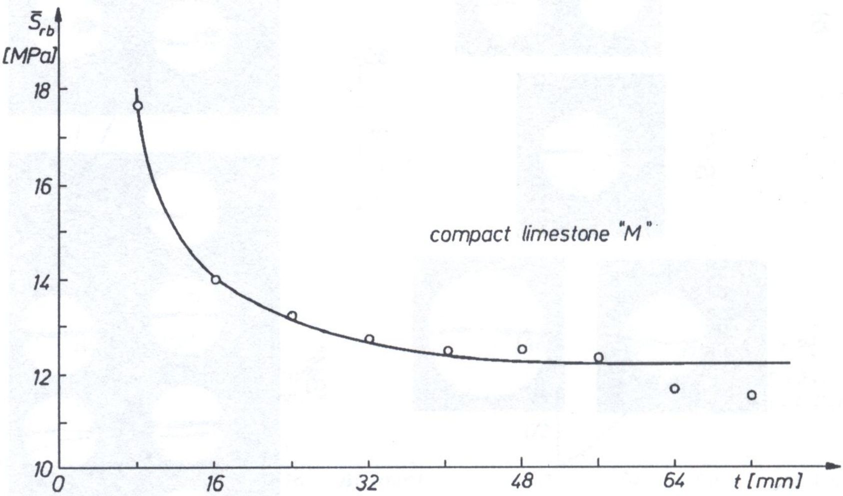

FIG. 6. Influence of the length (thickness) of a specimen on the value of S_b for limestone M.

3. On the causes of discrepancies between the strength S_r (uniaxial tension) and S_{rb} (Brazilian test)

It is known that there are many causes of discrepancies between S_r and S_{rb}. It appears that, depending on the boundary conditions for the loading, the dimensions of the tool and the specimen, the type of the material of the latter etc., one factor is usually decisive, the influence of the remaining ones being of less importance. Some of those causes will now be discussed.

3.1. The elastic model and the real characteristics of a brittle body

As already observed, the theory of the ordinary Brazilian test is based on the assumption that the material is perfectly elastic; however, the mechanical characteristics obtained by transverse compression tests do not justify such an assumption. As regards the examples illustrated in Fig. 4, this concerns not only the soft limestone P, but also the compact limestone M which shows, in general, linear elastic properties, similarly to the medium-grain marble BM. It is worthwhile to devote some attention to the characteristic of the compact limestone M subjected to an ordinary tensile test (Fig. 4 a). Contrary to the opinion often expressed in the literature concerning tensile characteristics of rocks it is, in the present case, of nonlinear type. Some segments of different curvatures can be discerned, which means changes in the structure occurring in the course of the loading process.

3.2. The state of stress, the place of initiation and the type of fracture

Application of the relation (1.2), which determines the approximate tensile strength is based on the assumption, in agreement with Griffith's theory, that the cracking process begins at the centre of the disc, where the principal stress σ_1, which is compressive, is three times as high as the principal (tensile) stress σ_2. It is obvious that in a two-axial compression-tension state, the limit tensile strength must not be identified with the uniaxial tensile strength S_r. Tests of brittle materials in such a two-axial state of streess show, however, that if the value of the ratio σ_1/σ_2 is low, the stress σ_2 does not differ very much from the uniaxial tensile strength. Figure 7 presents, after [8], diagrams of strength of concrete of various types confirming this fact. Also in a three-axial compression-tension state which corresponds, in the Brazilian test, to a plane state of strain, the strength of a brittle material is lower than that in a two-axial state of compression or uniaxial tension.

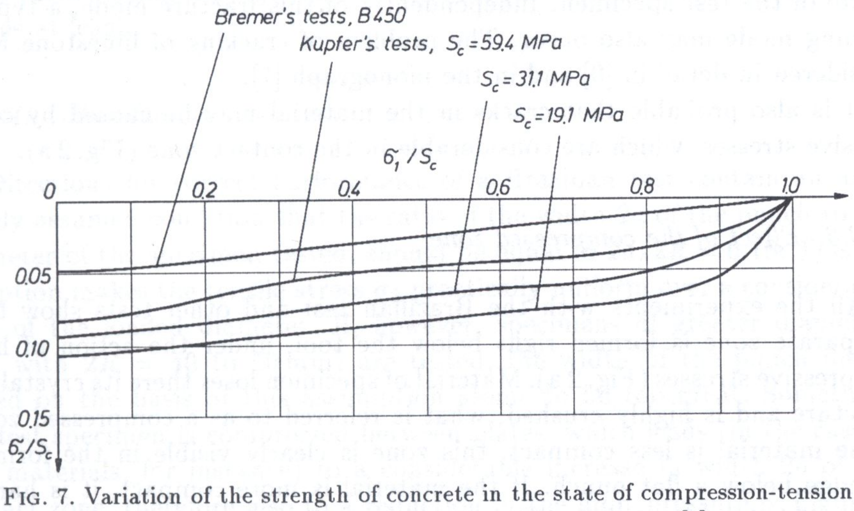

FIG. 7. Variation of the strength of concrete in the state of compression-tension (S_c – uniaxial compressive strength) according to [8].

However, the experimental data for the Brasilian test quoted in the literature and those given in Tables 2 and 3 of the present paper show an opposite trend, that is a higher strength σ_2 in the case of plane stress (flat specimens) and plane strain (specimens of greater length). Thus, the assumption that the cracks are initiated at the centre of the specimen does not appear to be correct. From experiments performed by various scientists including the present author [7], cracking in transversely loaded specimens is usually initiated in the neighbourhood of the region of contact between the specimen and the tool. The yield stress intensity in the material as evaluated according to Huber's theory in the case of plane stress or plane strain, and, according to the theory of maximum shear stress, is maximum at this particular place (Fig. 2 b). It is the place where the absolute value of principal stress σ_2 decreases, becomes zero on the diameter (the y-axis) and then, having already changed into compression stress, increases rapidly (Fig. 2 a). The tensile strength at the place of formation of a crack (which would correspond, in Fig. 7, to the points of curves of large values of the abscissae σ_1/S_c) is much lower than at the centre of the cross-section of the specimen, therefore it is probable that this is the reason of the value of the strength S_{rb} calculated for the centre being different (sometimes considerably) from that of S_r.

In the case of the Brazilian test, the usual type of fracture is that of separation caused by the action of tensile stresses, shearing modes of fracture occurring also in some cases, however. The latter form of cracking occurs in the case of compact limestone (limestone M) (Fig. 5 c). Cracking lines of complicated form originate at the edges of the punch and meet near the centre of the test specimen. Independently of this fracture mode, a typical opening mode may also occur. The problem of cracking of limestone M is considered in detail in [9] and in the monograph [7].

It is also probable that cracks in the material may be caused by compressive stresses, which are considerable in the contact zone (Fig. 2 a).

3.3. Effect of the compressed zone

All the experiments with the Brazilian test and other tests show that a separate zone is formed right below the tool, under the action of high compressive stresses (Fig. 2 a). Material of specimen loses there its crystalline structure and is highly crushed, what is referred to as a compressed zone. If the material is less compact, this zone is clearly visible in the form of a wedge below a flat punch. If the material is more compact, it is hardly distinguishable (Fig. 5 c). In the case of action of a wedge-shaped punch, such a zone is formed about it taking (for soft limestone, for instance) the form of a hemispherical core (Fig. 8). The compressed zone plays an important role in the process of cracking, because it is this particular region which transfers the load deeper into the material of the specimen.

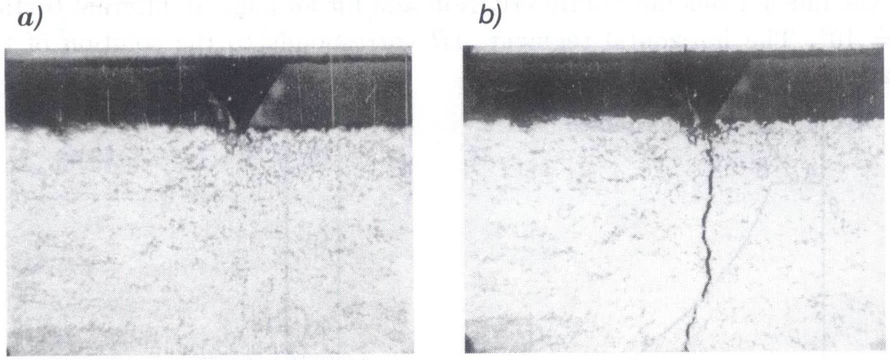

FIG. 8. The compressed zone around a wedge pressed against a test specimen of soft limestone (enlargement by several times) according to the present author's investigation: a) formation of a compressed zone, b) fracture of the specimen.

The strength properties of the compressed zone are unknown. It is probably occupied by a medium of nonlinear characteristic. It follows that we are concerned, in the Brazilian test, with interaction of materials of two kinds, having different properties, that is the compressed zone and the zone of intact material. The stress distribution at the contact between them is not known either.

The formation of a compressed zone results in the solutions of the elasticity theory being inadequate for the Brazilian test, in case of less compact bodies at least.

3.4. Other factors

Directions for correct performance of a Brazilian test contain an arbitrarily assumed condition that the ratio of the width $2a$ of the punch to the diameter of the specimen tested, should be equal to $2a/2R = 0.10$. This assumption makes the tensile stress σ_2 practically uniform over a considerable part of the loaded diameter. If, however, specimens of greater diameters (e.g. with $2R = 40$ to $50\,\text{mm}$) are tested, the width of the punch determined on the basis of this assumption seems to be too great. Sometimes the test specimen is compressed between plates, which leads (in the case of soft materials, for instance) to a considerable increase in width $2a$ of the contact zone, therefore also to a reduction in the limit pressure p. Figure 9 shows as an example, a diagram of load p referred to the cohesion c as a function of the ratio of the width of contact $2a$ to the diameter $2R$ (line ABC). This diagram is based on a theoretical solution, obtained by the method of characteristics, of the differential equations of limit equilibrium for the linear Coulomb failure criterion and for an angle of internal friction $\varrho = 10°$. The horizontal segment AB corresponds to the solution of the Prandtl problem [7].

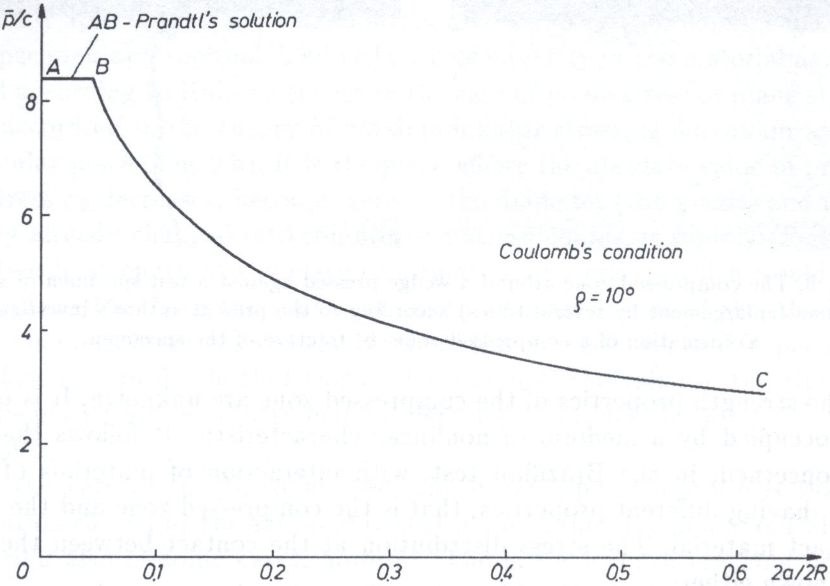

FIG. 9. Example of a diagram of boundary pressures p as a function of the ratio of the width of the contact zone to the diameter of the Brazilian test specimen for $\varrho = 10°$.

G. WIJK, who considered the Brazilian test in [3], draws a conclusion that discrepancies between S_r and S_{rb} may be caused by violation of the conditions of plane stress in the course of the loading process. By treating this test as a three-dimensional problem, he derived a relation expressing the stress σ_2 in the form of a function $\sigma_2 = f(x = 0, y, z)$, where z is half thickness of the test specimen: $z = t/2$. Figure 10 illustrates the character of variation of σ_2 according to the considerations contained in his paper for $z/R = 0.2$ and 0.3. It is seen that the tensile stress will be approximately uniform over the best part of the diameter for low values of z/R only, that is for values satisfying the condition $z/R < 0.2$; therefore it is seen that G. Wijk postulates the use of thin cylindrical specimens of thickness $t = 0.34R$.

However, satisfaction of this postulate would lead in practice to an excessive increase in S_{rb}, because compression of thin cylinders means influence of the scale effect (Fig. 6). As a consequence, it seems to be more reasonable to

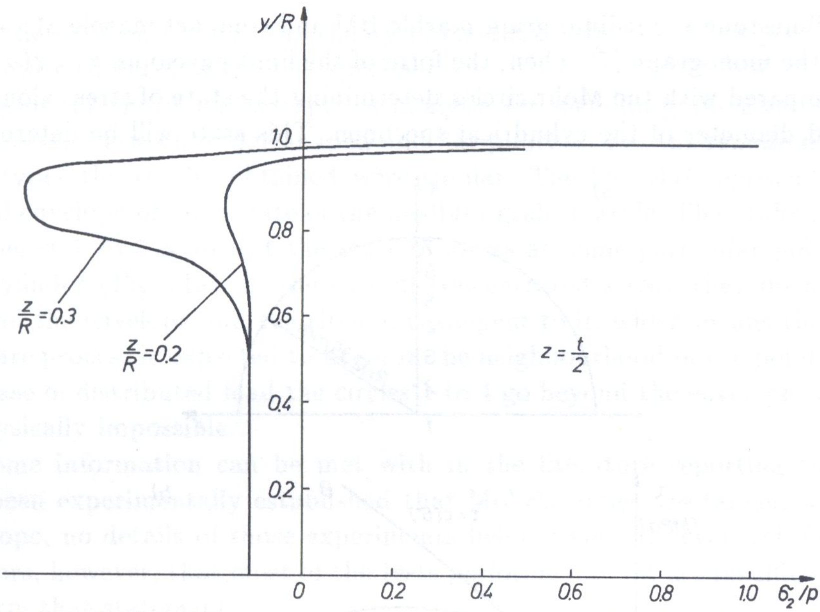

FIG. 10. Distribution of the stress σ_2 along the diameter as a function of the thickness of the test specimen, according to G. WIJK [3].

assume the length of the test specimen such that plane state of stress under compression is ensured. It is known that no analytical method is available for determining the limit value of $2a/t$ above which an approximately plane state would appear, therefore this value must be determined by experimental means (Fig. 6).

Another factor which may distort the results of calculation of S_{rb} (which will not be considered here, however) is most certainly the character of the stress distribution over the plane of contact. It is remembered that a uniform stress distribution was assumed in the theoretical analysis.

4. STATE OF STRESS AND THE REAL ENVELOPES OF LIMIT STATE

Knowing the causes of the discrepancies between S_r and S_{rb} which have just been discussed, we shall now verify whether the distribution of the stresses σ_1 and σ_2 along the loaded diameter of the cylinder predicted by the theory of elasticity is realistic, that is whether the condition of limit state $F(\sigma_{ij}) = 0$ (σ_{ij} denoting the stress tensor of limit state) has been exceeded or not. To this aim use will be made of the experimental results discussed in Sec. 2 and the actual envelopes of the limit state $\tau = \tau(\sigma)$ (τ, σ denoting the shear and normal stress, respectively, in the slip plane) for the rocks tested (soft limestone P, medium-grain marble BM and compact marble M), taken from the monograph [7]. Then, the form of the limit envelopes $\tau = \tau(\sigma)$ will be compared with the Mohr circles determining the state of stress along the loaded diameter of the cylindrical specimen. This state will be determined

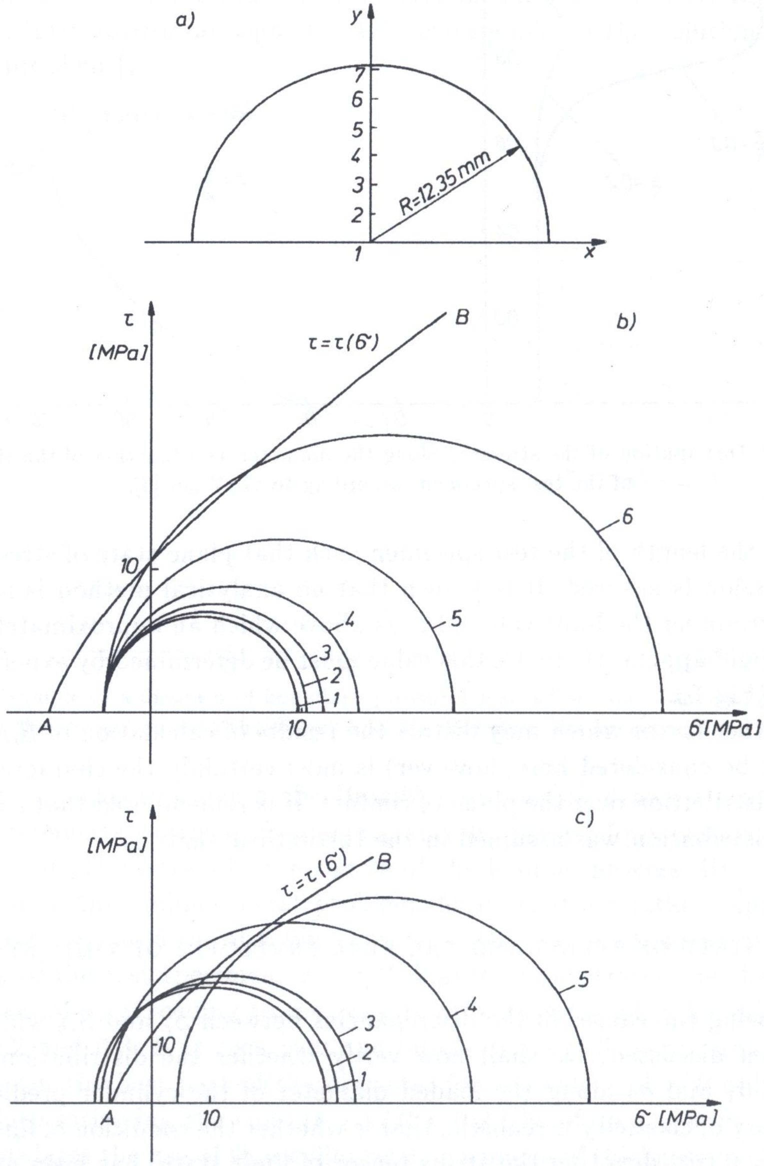

FIG. 11. Mohr's circles (solution by methods of the theory of elasticity), against the background of the actual envelope of limit state of medium-grained marble BM; a) points of the diameter of the test specimen for which the state of stress (fracture) has been determined, b) loading by wedges, c) loading by flat punches.

from the relation (1.1) of the theory of elasticity expressing the pressure p of fracture (failure) of the test specimen.

Figure 11 shows an example of analysis for medium-grain marble BM compressed between wedge-shaped and flat punches. For the remaining two rock types the results obtained were similar. The line AB represents the actual envelope of limit state of the medium-grain marble. The Mohr circles numbered 1 to 6 represent the state of stress at some particular points of the cylinder (Fig. 11a). In the case of concentrated stress, they do not go beyond the envelope and the circle 6 is tangent to it, which means that the fracture process is expected to begin in the neighbourhood of the point 6. In the case of distributed load the circles 1 to 4 go beyond the envelope, which is physically impossible.

Some information can be met with in the literature reporting that it has been experimentally established that Mohr's circles are tangent to the envelope, no details of those experiments being given, however, (cf. [6]). It appears, however, that most of the tests performed as yet are insufficient to confirm that statement.

5. The Brazilian test as a problem of theory of limit state

The Brazilian test may also be treated as a problem of the theory of limit state. This possibility has already been observed by P. Habib, D. Radenković and J. Salençon [10], who assumed Coulomb's criterion as a condition of failure, the expected mode of fracture being, in most cases, in disagreement with the actual mode of cracking.

The only exceptional case is that of compact limestone, in which real cracking lines qualitatively comply with the theoretical lines of displacement rate obtained under the Coulomb condition and the non-associated law of flow (Fig. 5 c M). This type of fracture is explained in [9].

5.1. Experimental results: compression of cylindrical and rectangular specimens

Cylindrical test specimens of soft limestone P, medium-grain marble BM and compact limestone M were compressed between rigid plates, the loading scheme thus being the same as that often used during the ordinary Brazilian test. Some scores of experiments were performed. The limit pressure p (referred to the cohesion c) is represented in Fig. 12 as a function of R/a, by points marked against the background of theoretical solutions obtained by assuming three different limit state conditions, namely that of Mohr and the modified and non-modified Coulomb condition. It is seen that those points lie in the neighbourhood of the solutions obtained for the first two conditions.

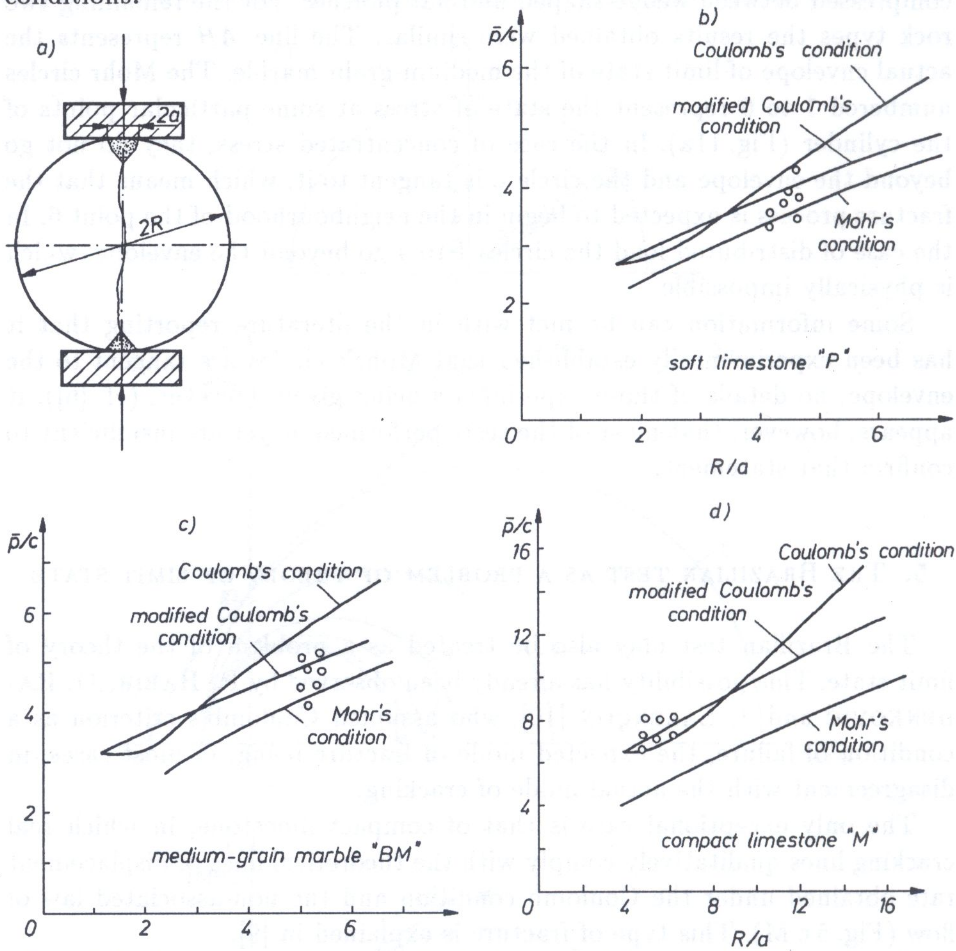

FIG. 12. Limit pressures p/c against the background of theoretical solutions, based on the Mohr condition and the modified and non-modified Coulomb condition; a) loading scheme, b) soft limestone P, c) medium-grain marble BM, d) compact limestone M.

Similar tests were made, consisting in transverse loading of blocks in the form of rectangular prisms. The blocks were made of soft and compact limestone P and M, and of marble BM, the loading was applied according to the scheme in Fig. 1 c. Plane state of stress was produced. As an example, the theoretical results obtained under the Mohr condition and the modified Coulomb condition are confronted in Fig. 13 with the experimental results obtained for soft limestone.

Those results confirm the inference that the Mohr condition and the modified Coulomb condition describe in a fairly realistic manner the behaviour of the known three types of rocks subject to the action of the limit load.

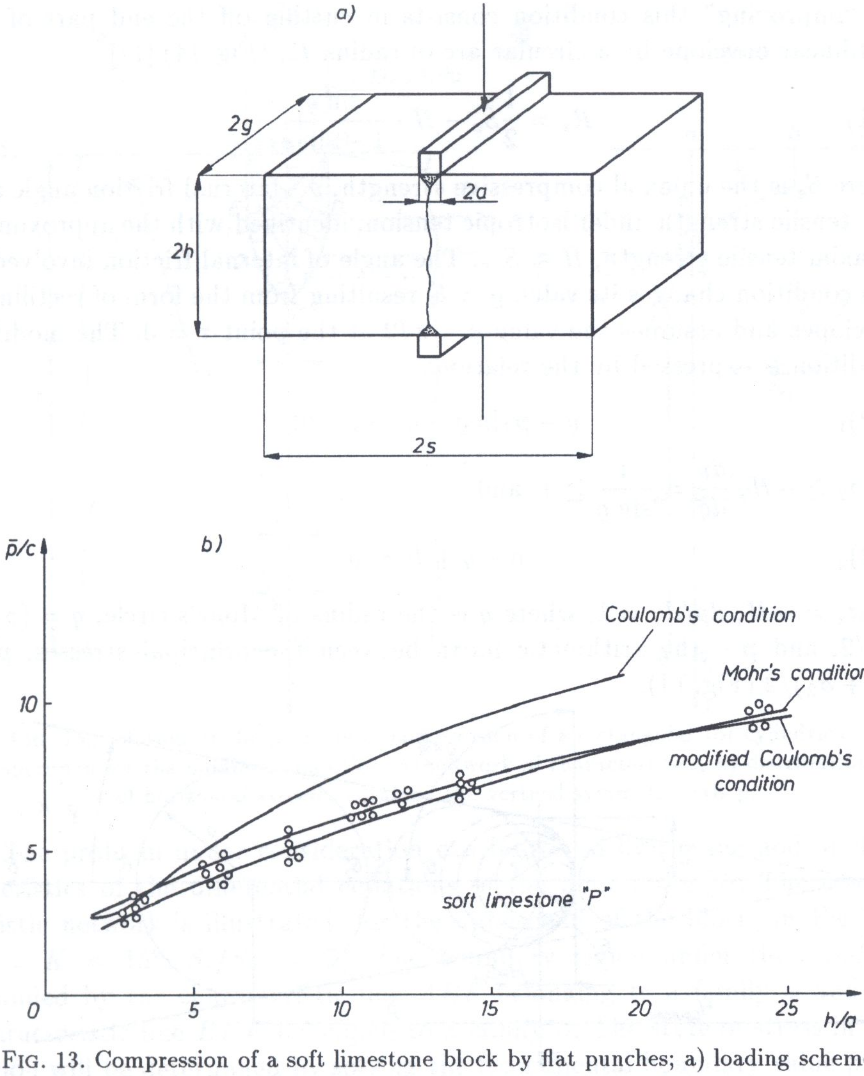

FIG. 13. Compression of a soft limestone block by flat punches; a) loading scheme and mode of cracking, b) verification of the theoretical solutions.

The modified Coulomb condition anticipates well the mode of fracture of the specimens tested, which is that of two wedge-shaped regions appearing under the punches, connected by a plane vertical crack (Figs. 12 a and 13 a). This is the reason, for which this condition will be discussed in greater detail.

5.2. Modified Coulomb condition

It is known that the linear Coulomb condition yields a value of the tensile strength which exceeds too much the real value. One of the methods for "improving" this condition consists in cutting off the end part of the rectilinear envelope by a circular arc of radius R_z (Fig. 14) [11]

$$R_z = \frac{1}{2}S_c - H \cdot \frac{\sin \varrho}{1 - \sin \varrho}, \tag{5.1}$$

where S_c is the uniaxial compressive strength, ϱ – internal friction angle and H – tensile strength under isotropic tension, identified with the approximate uniaxial tensile strength, $H \approx S_{rz}$. The angle of internal friction involved in this condition changes its value $\varrho = K$ resulting from the form of rectilinear envelopes and assumes the value $\varrho = \pi/2$ at the point $\tau = 0$. The modified condition is expressed by the relations

$$(5.2)_1 \qquad q - p \sin \varrho - c \cos \varrho = 0,$$

for $\sigma_2 \geq -H$, $\dfrac{dp}{dq} = \dfrac{1}{\sin \varrho} \geq 1$, and

$$(5.2)_2 \qquad p - q + H = 0,$$

for $\sigma_2 = -H$, $dp/dq = 1$, where q is the radius of Mohr's circle, $q = (\sigma_1 - \sigma_2)/2$, and p – the arithmetic mean between the principal stresses, $p = (\sigma_1 + \sigma_2)/2$ (Fig. 14).

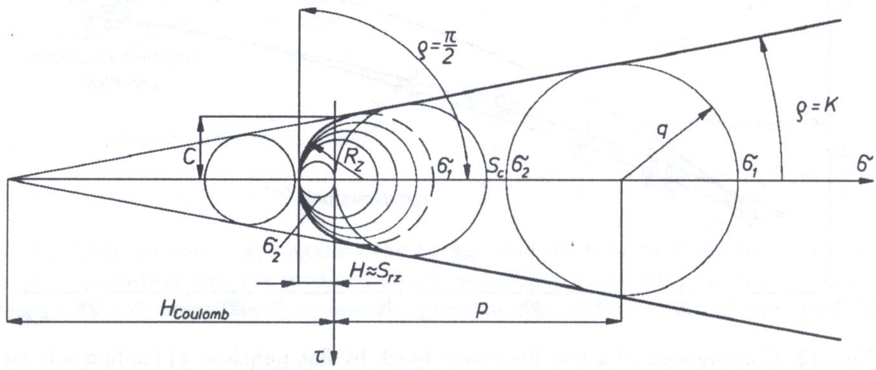

FIG. 14. The modified Coulomb condition.

5.3. Analysis of the Brazilian test with the modified condition

Accurate analysis of the problem of a block having the form of a rectangular prism (which can be generalized to the problem of a cylindrical block) compressed by flat punches was made in the present author's papers [7, 12] assuming the original and modified Coulomb conditions and a state of plane strain. We are now going to explain the most important points of that analysis, disregarding the mathematical relations.

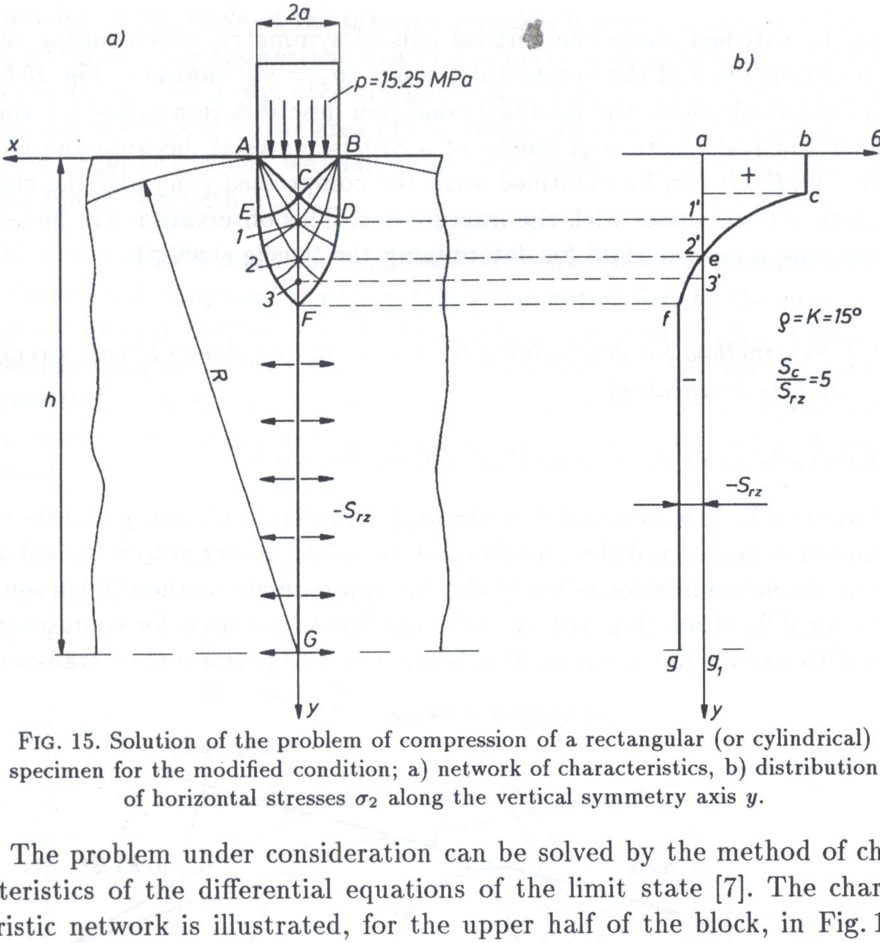

FIG. 15. Solution of the problem of compression of a rectangular (or cylindrical) specimen for the modified condition; a) network of characteristics, b) distribution of horizontal stresses σ_2 along the vertical symmetry axis y.

The problem under consideration can be solved by the method of characteristics of the differential equations of the limit state [7]. The characteristic network is illustrated, for the upper half of the block, in Fig. 15 a ($\varrho = K = 15°$, $S_c/S_{rz} = 5$). The boundary region under the punch is bounded by the characteristic line AEF belonging to a family α and the characteristic line BDF belonging to a family β. The state of stress in this region will be determined by solving the fundamental boundary value problem, use being made of relations describing the characteristics which result from the linear Coulomb condition ($\varrho = K =$ const). Below the region $AEFDB$ we have a single characteristic FG, which is a consequence of the fact that the limit state is now described by the relations $(5.2)_2$, therefore the differential equations of the limit state are parabolic. The stresses acting along FG are those of tension, $\sigma_2 = -S_{rz}$ and compression σ_1 (the circle of radius R_z in Fig. 14). F is a singular point where the internal friction angle changes its value $\varrho = K$ for $\varrho = \pi/2$.

The integral equilibrium condition of the block $\int_0^h \sigma_2\, dy = 0$ must of course be satisfied along the vertical axis of symmetry, which means that the resultant force of the horizontal stresses ($\sigma_x = \sigma_2$) vanishes (Fig. 15 b).

As observed above, the modified condition describes in a rather accurate manner the real fracture of blocks of a brittle material, because the limit region $AEFDB$ can be identified with the compressed zone and the characteristic FG coincides with the fracture line. This observation will be used for devising a new method for determining the tensile strength.

5.4. New method for determining the approximate value of tensile strength and its verification

Two variants of the new method will be discussed.

VARIANT I. It is assumed that the angle of internal friction ϱ (within the compression range) and the cohesion c of the brittle material to be tested are known; the determination of both (also the approximate method [7]) presents no major difficulties (Fig. 16). Rectangular blocks are used for compression tests with narrow flat punches, this being easier to perform than transverse

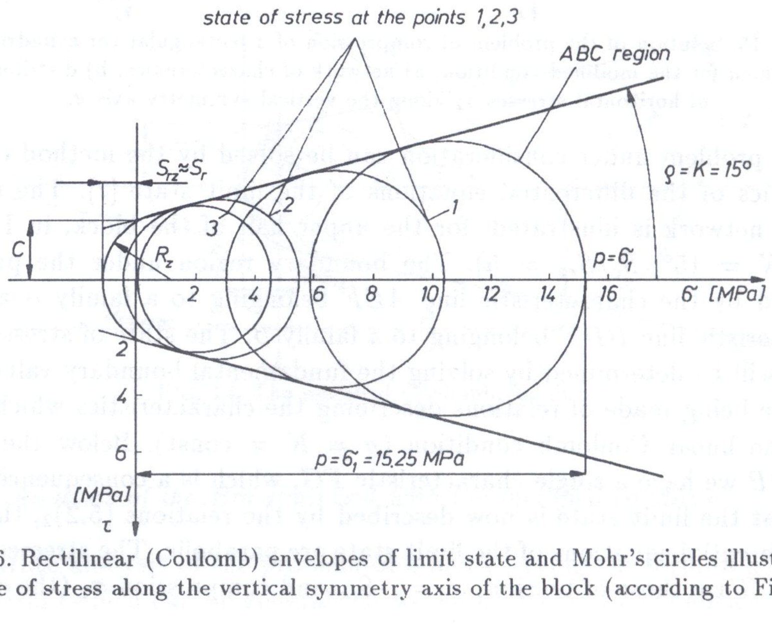

FIG. 16. Rectilinear (Coulomb) envelopes of limit state and Mohr's circles illustrating the state of stress along the vertical symmetry axis of the block (according to Fig. 15 a).

compression tests of cylindrical specimens. The limit pressures p and their mean values are calculated for several experiments. Further procedure is based on the use of the characteristic network (resembling that of Fig. 15 a) for a particular value of h/a and a diagram of $\tau = \tau(\sigma)$ (Fig. 16). The knowledge of the value of the pressure p makes it possible to determine the state of stress along the vertical axis of symmetry within the region $AEFDB$ (Fig. 15 a), and the region $abce$ of compressive stress σ_2 (Fig. 15 b). From the condition of equality of the area of this region and the area of the region of tension $efgg_1$ (Fig. 15 b), we shall obtain the sought – for approximate value of the tensile strength S_{rz}.

VARIANT II. We mark the experimental points for a few values of h/a on the theoretical diagrams of $p/c = f(h/a)$ corresponding to the prescribed angle of internal friction $\varrho = K$, but various values of S_c/S_{rz}. Figure 17 presents examples of such diagrams for an angle $\varrho = 31°$ (medium-grain marble BM) and $S_c/S_{rz} = 5$ and 10. From the form of the experimental curve further inferences can be drawn as regards approximate value of tensile strength.

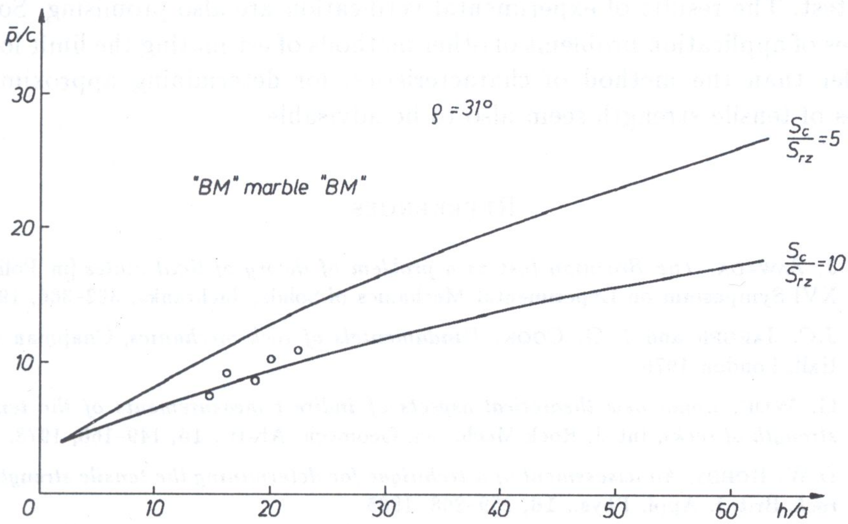

FIG. 17. Theoretical diagrams of $p/c = f(h/a)$ for internal friction angle $\varrho = 31°$ and experimental points, the location of which can be used to determine S_{rz}.

The correctness of the new method has been verified for the Variant I, the results obtained being as follows.

Soft limestone P, $h/a = 8.3$, $S_{rz} = 1.8$ MPa;

medium-grain marble BM, $h/a = 14.7$, $S_{rz} = 5.3$ MPa;

compact limestone M, $h/a = 24.0$, $S_{rz} = 6.9$ MPa.

The tensile strength S_{rz} approaches now better the ordinary tensile strength than the result of the Brasilian test (Table 3). The coefficient $n_2 = S_{rz}/S_r$ takes the following values:

$n_2 = 1.01$ for soft limestone P,
$n_2 = 1.09$ for medium-grain marble BM, and
$n_2 = 0.6$ for compact limestone M.

6. Concluding remarks

From the considerations presented it follows that the theory of the Brazilian test based on the elastic model does not describe in an accurate manner the real behaviour of the material under the action of a load. It is possible that this description could be improved by some modifications, e.g. by taking into consideration the effect of the compressed zone, or the non-uniform character of pressure distribution over the surface of contact.

The way of considering the Brazilian test as a problem of the theory of limit state appears to contribute to a certain progress in the analysis of that test. The results of experimental verification are also promising. Some studies of application problems of other methods of estimating the limit load, simpler than the method of characteristics, for determining approximate values of tensile strength seem also to be advisable.

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