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DRY FRICTION SELF-EXCITED VIBRATIONS ANALYSIS AND EXPERIMENT

R. BOGACZ and B. RYCZEK (WARSZAWA)

The paper is devoted to the experimental and theoretical analysis of a set composed of several degrees of freedom mass-system interacting with a moving belt by means of dry friction. The experimental study allows to formulate the friction model depending on velocity of motion, acceleration, time of adhesion and force rate.

1. Introduction

The development of friction-induced vibrations theory as a part of nonsmooth dynamical system theory has become increasingly important in engineering. Promotion of high speed tracked transportation systems required extensions for analytical and numerical methods of analysis of grating brakes and wheel/rail drive systems. There are numerous examples of friction-excited vibrations in engineering systems as well as in everyday life [1, 2]. Sometimes the friction-sustained vibrations are undesired and should be avoided because of noise and wear, but the same kind of oscillations are used in bowed instruments. The wheel/rail contact problem is modelled in [3] taking into account non-linear, nonsmooth friction law to explain wavy shape of rail wear – corrugations (Figs. 1 a and 1 b), e.g. [4]. Depending on the friction model, the rolling moment oscillates in time qualitatively different, what is visible in Fig. 2. The method of discretisation influenced the results. Therefore, there is a need for theoretical and experimental investigations which enable to distinguish numerical and physical effects.

Some relatively simple models of friction were examined in a system of one degree of freedom in order to divide the velocity-damping plane into regions with qualitatively similar solutions [5] or investigate the influence of external excitation on the stability and bifurcation of solutions [6]. The aim of this study is to examine a more complicated model of friction. In our investigations, the assumption is made that the friction force depends on the following parameters:

- velocity of relative motion,
- acceleration,
- time of stick (adhesion),
- gradient of the friction force change during slip-stick phase.

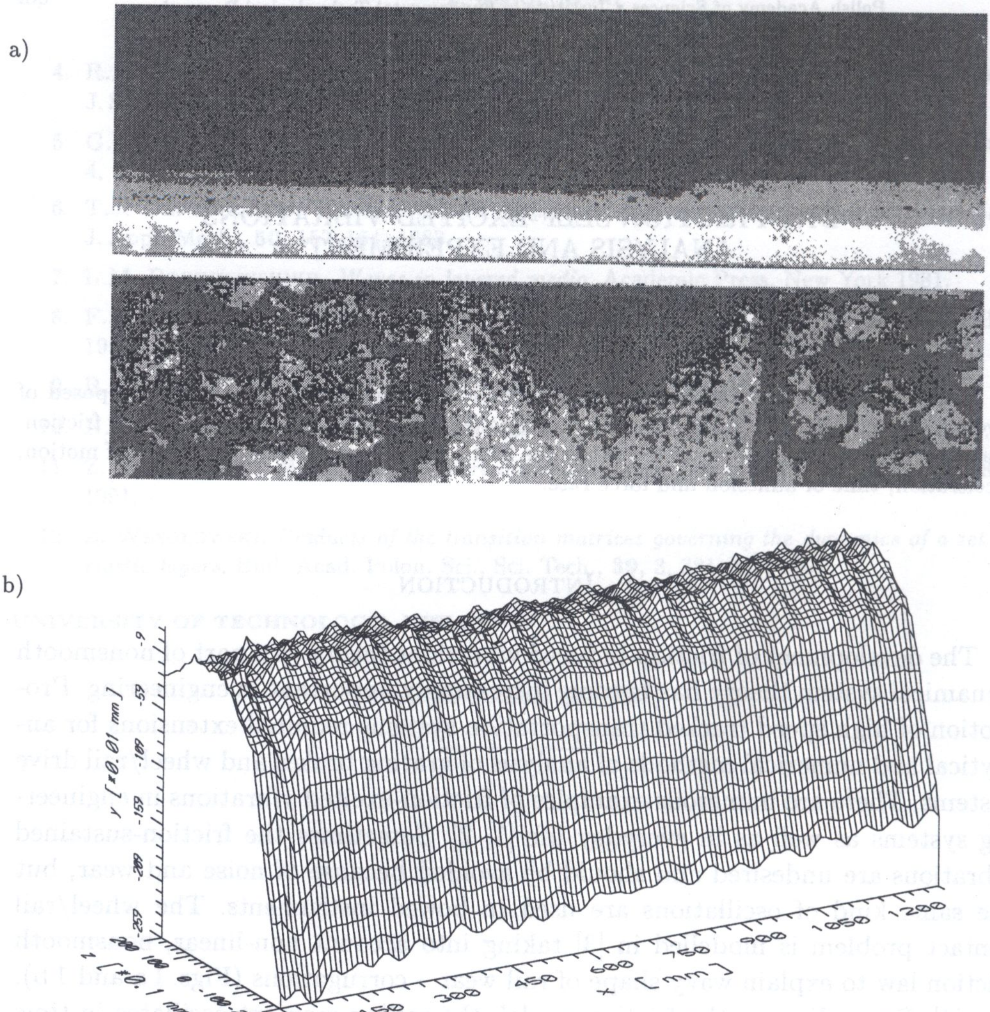

FIG. 1. Corrugated rail: view (a) and results of measurements (b).

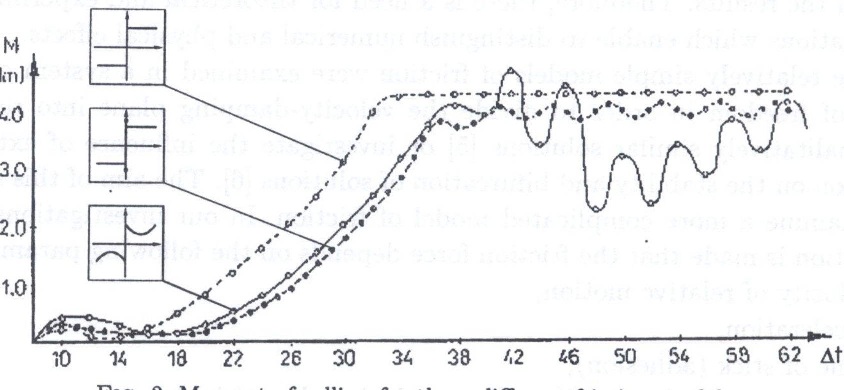

FIG. 2. Moment of rolling for three different friction models.

Such phenomena as the dependence of the friction force on normal stress, roughness, temperature and many others are not included in this consideration.

The optical measurement system was used in the experimental investigations. The system of analog to digital conversion and data recording made it possible to identify the proposed model of friction for steel–polyester pair.

2. Model of mechanical system

Let us consider a mechanical system (see Fig. 3), consisting of a few masses, connected by linear, noninertial springs, resting on the belt moving at a constant velocity. Between the belt and the masses occurs a friction force F_n.

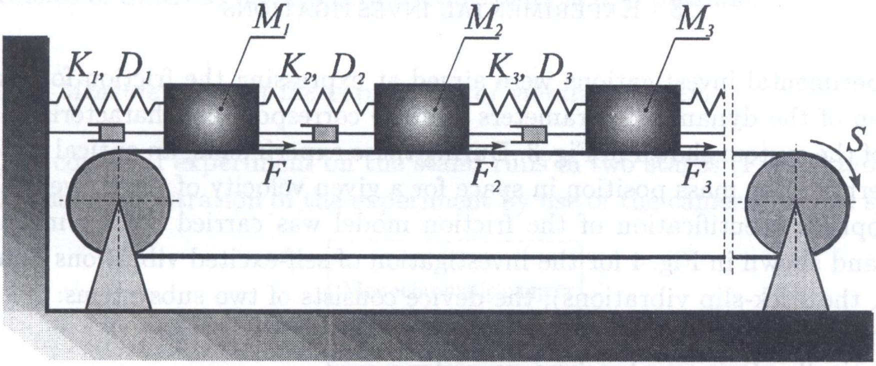

FIG. 3. Scheme of n-degrees of freedom system with dry friction.

We assume that the state of equilibrium is defined by such a position of masses, where the springs are free (not under tension). We also assume, that the function representing the friction model is dependent on relative velocity, odd, continuous and piecewise linear.

Motion of the investigated system for $n = 1$ is described by the following equation:

$$MX'' + DX' + KX = F(W), \tag{1}$$

where X – mass deflection from the state of equilibrium, K – spring stiffness, D – viscous damping, F – friction force, τ – time, $S > 0$ – belt velocity,

$$X' = \frac{dX}{d\tau}, \qquad W = S - X'.$$

After assumption of the following dimensionless quantities:

$$\begin{aligned} t &= \frac{\tau}{\tau_0}, \qquad x = \frac{KX}{F_0}, \qquad d = \frac{D}{\sqrt{KM}}, \\ s &= \frac{S\sqrt{KM}}{F_0}, \qquad f(w) = \frac{F(W)}{F_0}, \end{aligned} \tag{2}$$

where

$$\dot{x} = \frac{dx}{dt}, \qquad \tau_0 = \sqrt{\frac{M}{K}},$$

F_0 – friction unit (e.g. body weight or normal force), τ_0 – time unit; equation of motion (1) takes the following form:

$$\ddot{x} + d\dot{x} + x = f(w), \tag{3}$$

where $f(w)$ – function describing the friction model, d – dimensionless damping, w – dimensionless relative velocity, $w = s - \dot{x}$, s – dimensionless belt velocity.

3. Experimental investigations

Experimental investigations were aimed at expressing the friction force as a function of the dynamical parameters and the corresponding characteristic features of the system shown in Fig. 3. During these experiments, an optical method to determine the mass position in space for a given velocity of the conveyor belt was applied. Identification of the friction model was carried out by means of test-stand shown in Fig. 4 for the investigation of self-excited vibrations (among others, the stick-slip vibrations); the device consists of two subsystems:

- mechanical,
- optically-electronical measuring system.

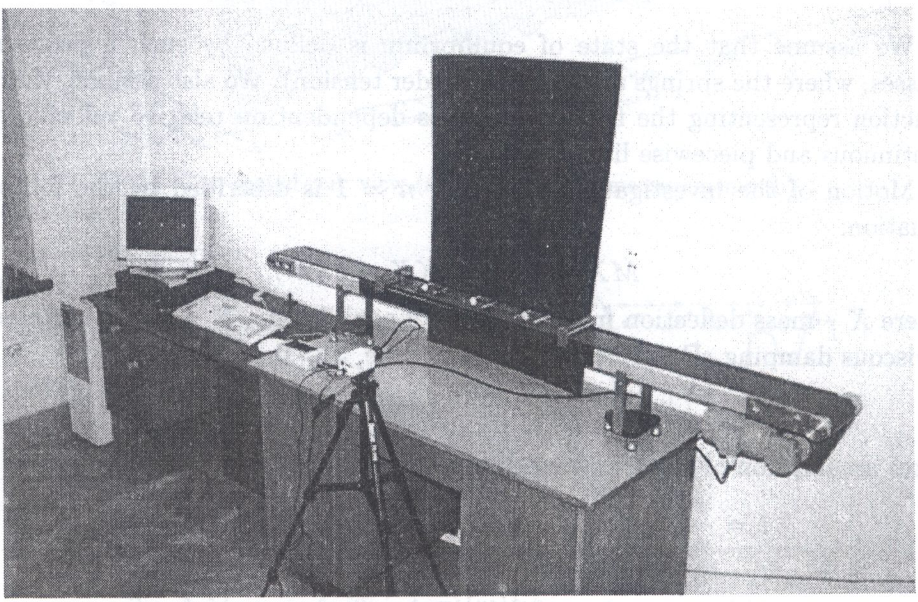

FIG. 4. Test-stand for investigation of discrete systems with dry friction.

The mechanical system is composed of masses connected by a springs system interacting with the conveyor belt. For the measurement we use a system of visualisation, acquisition and handling of the data. The analog-to-digital converter card for transformation of the video signal into digital form is applied in this system.

The stand enables the investigation of vibrations for chosen values of the following parameters:

- number of masses $i_m = 1, 2, 3$,
- velocity of the belt S,
- initial conditions X_0, $\dot{X}_0$,
- stiffnesses K_n, masses M_n.

Choice of different materials of friction pairs is also possible.

3.1. Measurements of the parameters of vibrating system

The course of experiment on the stand runs in two stages (Figs. 5 and 6):
1) analog registration of the experiment by use of the camera – video system,

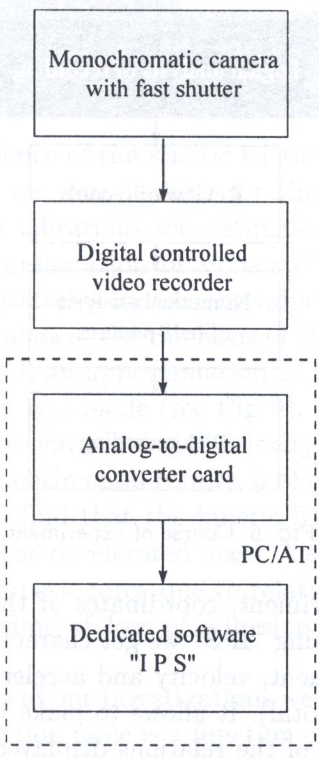

FIG. 5. System of visualisation, acquisition and handling of data.

2) computer analysis of the recording by means of the dedicated software "IPS" – Image Processing System.

The use of the analog registration of the experiment (on magnetic tape) allows to carry out the digital analysis recorded just after the registration or later.

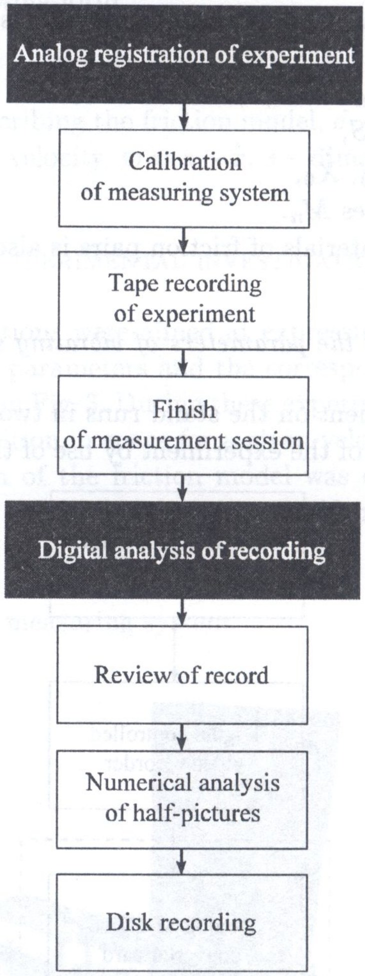

FIG. 6. Course of experiment.

As a result of the experiment, coordinates of the mass position during the motion can be obtained. Using "IPS" we get characteristics of the mass motion: time functions of displacement, velocity and acceleration, phase plane, acceleration spectra (actual and total). It allows to make initial analysis of vibrations of the system. An example of the relations displayed on the monitor screen are shown in Fig. 7.

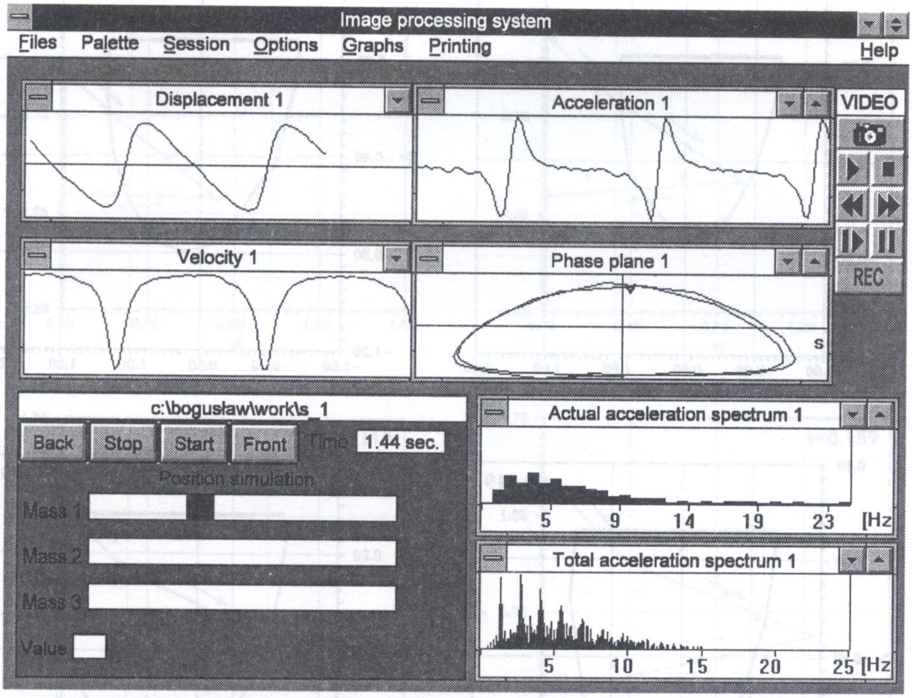

FIG. 7. Characteristic parameters of system vibrations displayed on the monitor screen by use of "IPS".

To determine the dependence of the kinetic friction force as a function of the relative velocity of motion, we were looking for the dynamical characteristics of the system. Periodicity of vibrations was estimated using the function based on the idea of the first representation of Poincaré [5]. In Fig. 8 examples of experimentally obtained trajectories for various values of belt speed for periodic vibrations of the system are shown. On the basis of the obtained characteristics, using the least square method, an approximation of the kinetic friction force as a function of relative velocity was made (see Fig. 9).

The kinetic force as a function of relative velocity for various belt velocities, after approximation of the experimental results, is shown in Fig. 10. On the basis of the above results one can find that the kinetic friction force for accelerated mass is higher than that for the decelerated mass. Also with higher belt velocity, increases both value of the kinetic force due to relative velocity values, and the static friction force at the time of loss of adhesion on the contact surface of mating bodies.

On the basis of the results of our investigations we can assume that the maximum value of the statical friction force is a function of the following parameters:

$$F_s = F_s(\tau_s, \Delta), \tag{4}$$

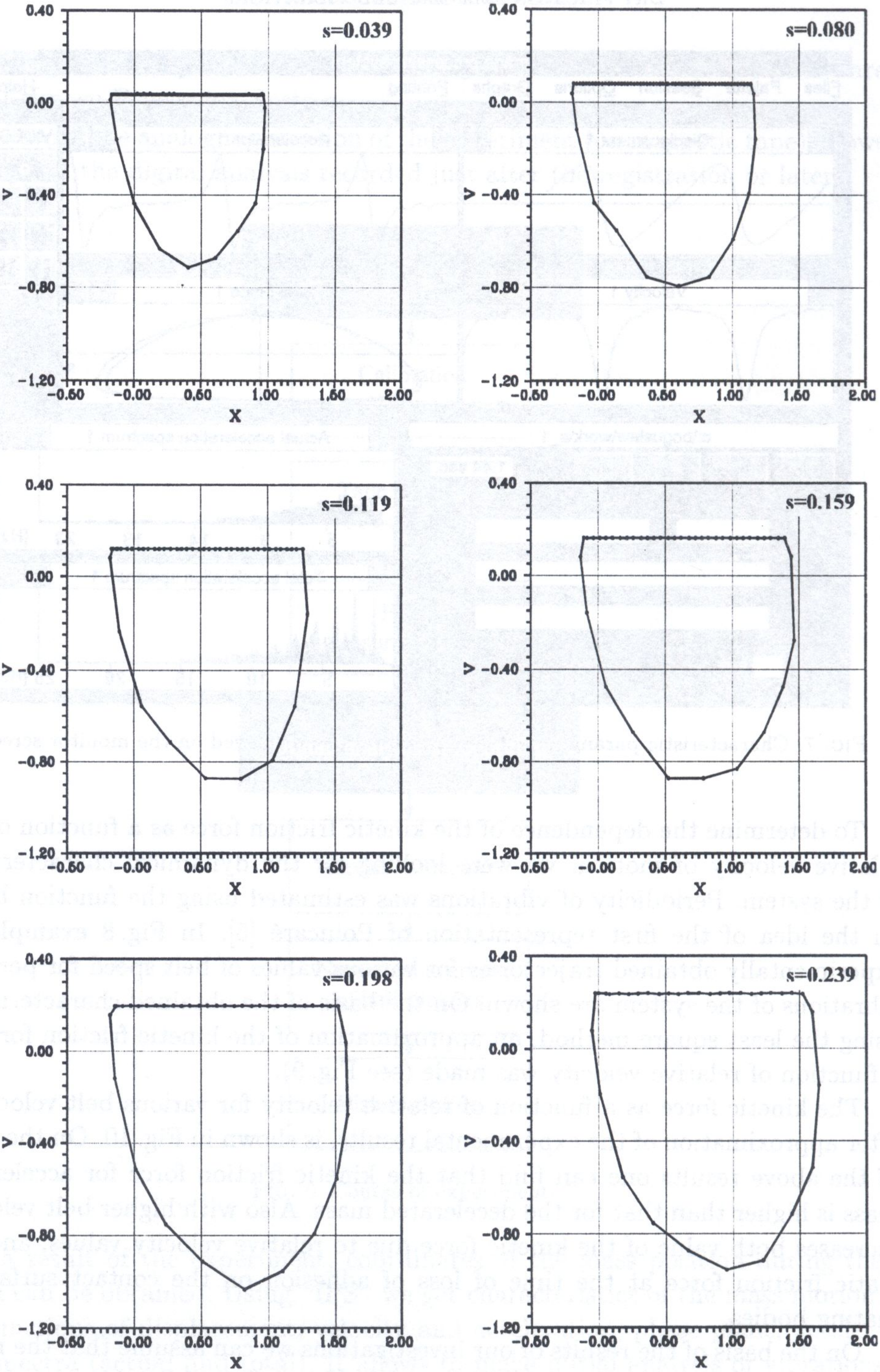

FIG. 8. Experimentally obtained phase trajectories of mass motion for various values of belt velocity s.

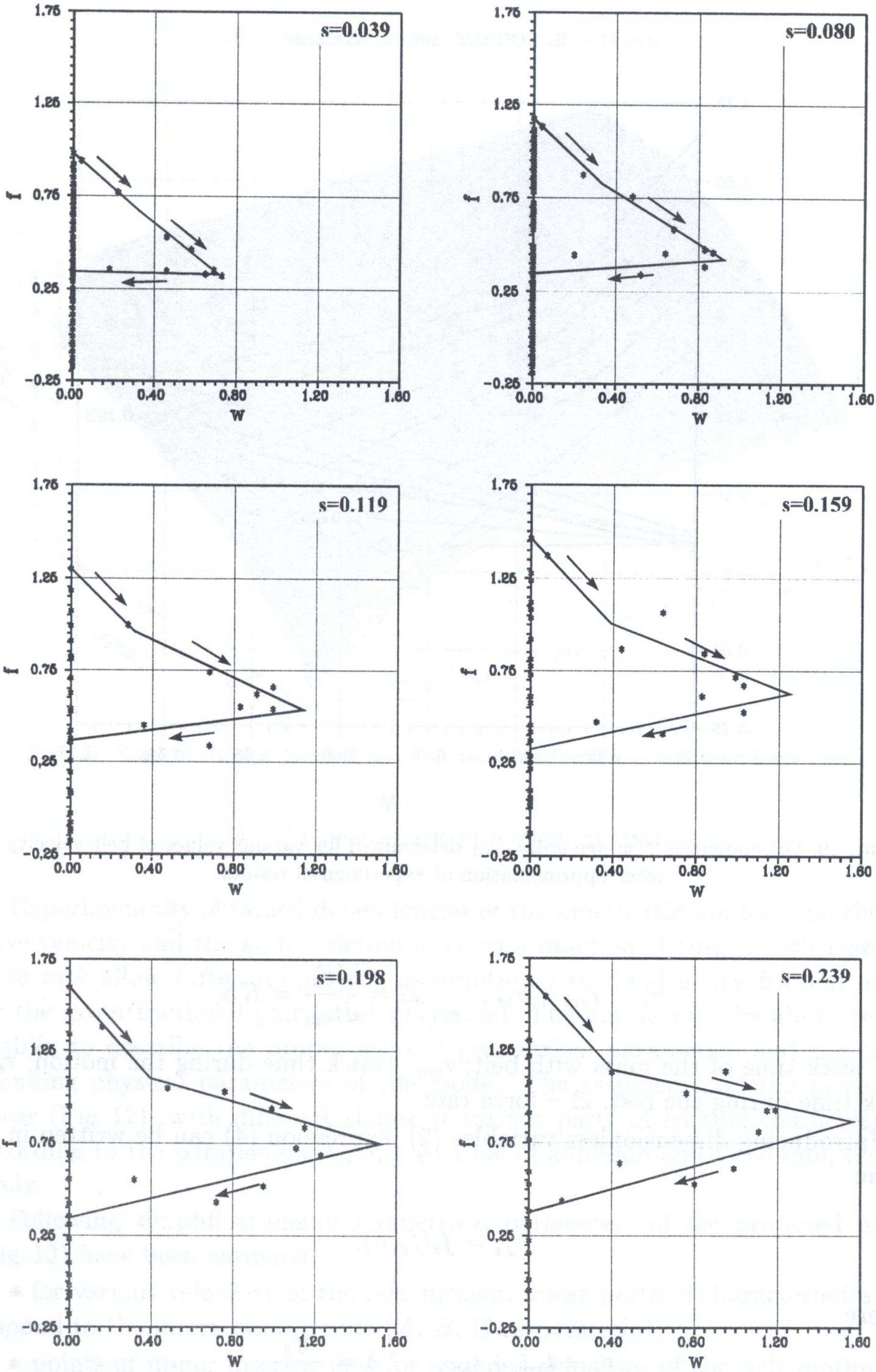

FIG. 9. Approximation of kinetic friction force f versus relative velocity w, for various values of belt velocity s.

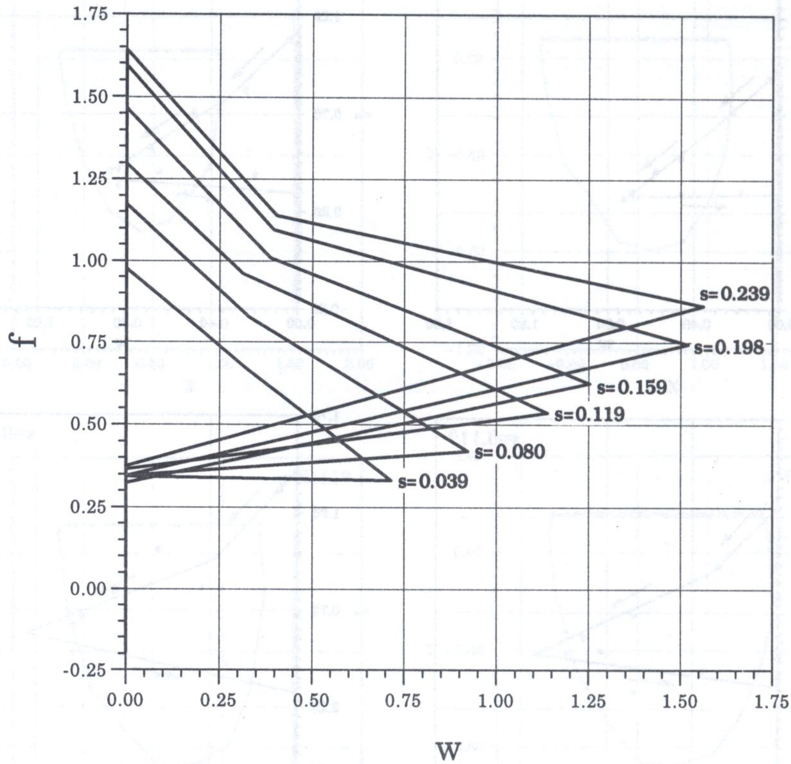

FIG. 10. Dependencies $f(w, \operatorname{sgn} \dot{w})|_{s=\mathrm{const}}$ determined for various values of belt velocity s after approximation of experimental results.

where

$$\tau_s = \tau_{sm} + \tau_{sr}, \qquad \Delta = \frac{dF_s}{d\tau_{sm}} = KS,$$

τ_s – stick time of the mass with belt, τ_{sm} – stick time during the motion, τ_{sr} – stick time during the rest, Δ – force rate.

Introducing dimensionless variables (2), expression (4) can be written in the form:

$$f_s = f_s(t_s, \delta), \tag{5}$$

where

$$t_s = t_{sm} + t_{sr}, \qquad \delta = \frac{df_s}{dt_{sm}}.$$

The dependence (5) has been experimentally determined and the results are plotted in Fig. 11.

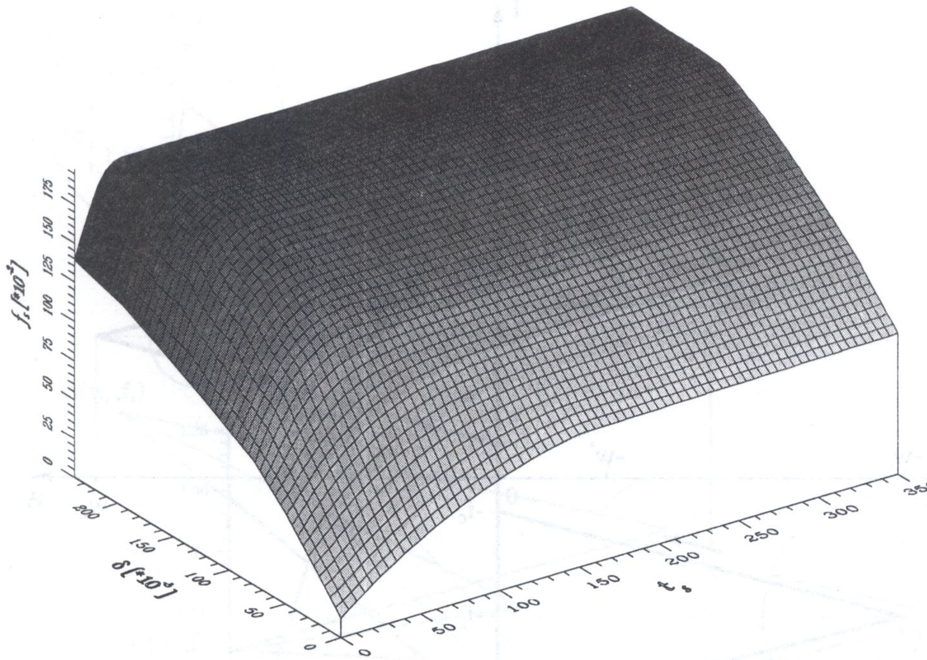

FIG. 11. Statical friction force f_s as a function of adhesion time t_s and force rate δ.

4. Proposal of friction model

Experimentally obtained dependencies of the kinetic friction force on the relative velocity and the static friction force as a function of time of adhesion and force rate allow (after simplifying assumptions) to build a dry friction model for the given frictional pair (steel–polyester). The analysis of the above results enables to describe the properties, the geometrical parameters and the corresponding physical parameters of the model. The assumed model is piece-wise linear (Fig. 12), with different slopes of various parts of friction characteristic, according to the parameters: t_s, δ, i. e.: time of adhesion and force rate, respectively.

Following simplifications of geometrical parameters of the proposed model (Fig. 13) have been assumed:

- for various velocities of the belt motion, linear parts of characteristics correspond to the intersection points (A, B, C respectively),
- points of upper intersections for assigned velocities of the belt motion are on a straight line (l),
- for given velocities of the belt motion, coordinates $w_{\max} = w_k$ have been obtained from the condition $\dot{w} = 0$.

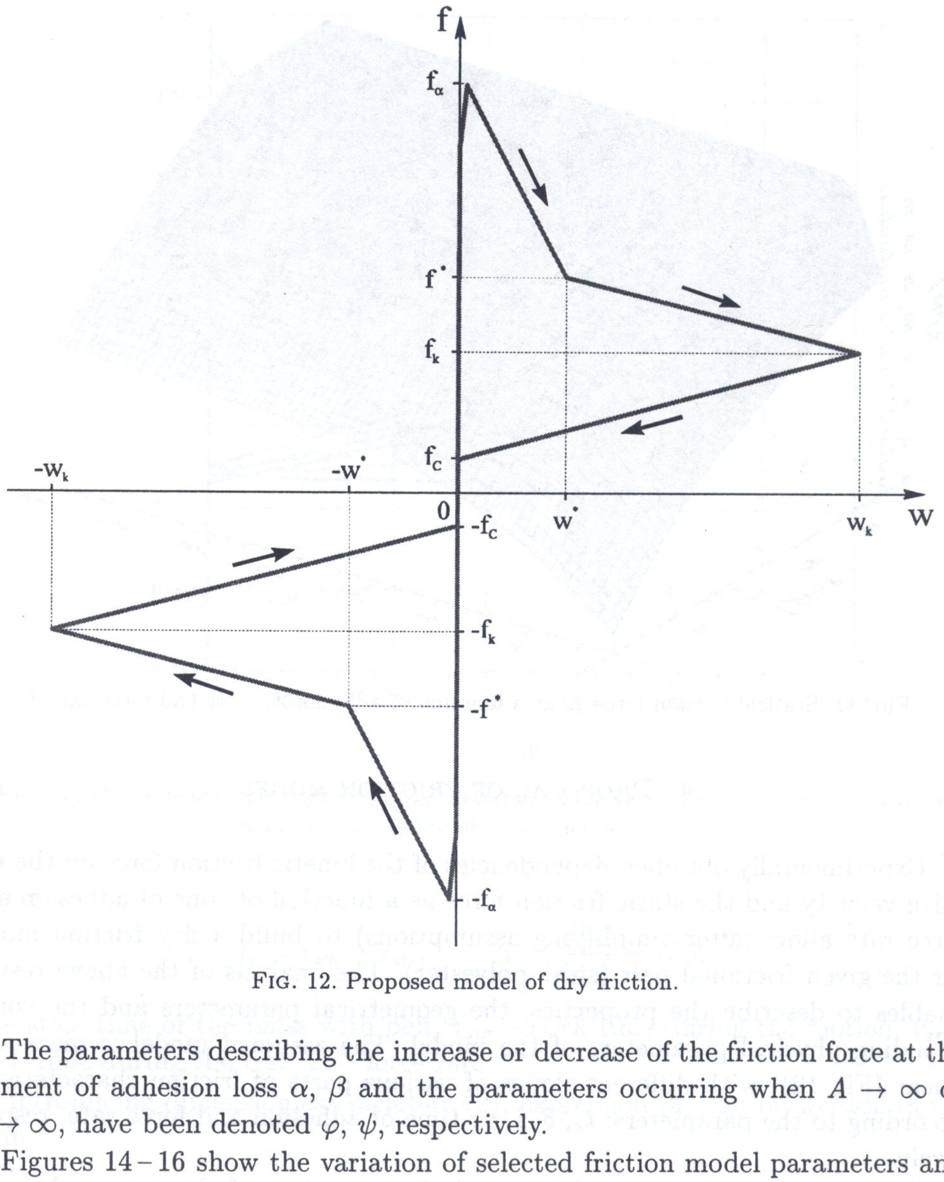

FIG. 12. Proposed model of dry friction.

The parameters describing the increase or decrease of the friction force at the moment of adhesion loss α, β and the parameters occurring when $A \to \infty$ or $C \to \infty$, have been denoted φ, ψ, respectively.

Figures 14 – 16 show the variation of selected friction model parameters and its influence on the phase trajectories of system motion for assigned values of velocities of belt motion and initial conditions. Translation of point C and the corresponding different shapes of trajectories are shown in Fig. 14. Another form of trajectories one can obtain in the case of rotation of straight line l around the point $O(0.35; 0.9)$, what is shown in Fig. 15. Phase trajectories for various values of parameter α, also show differences (see Fig. 16). The coordinates of point O and the parameters denoted in figures by index 0, correspond to the values assumed on the basis of results of experimental investigations.

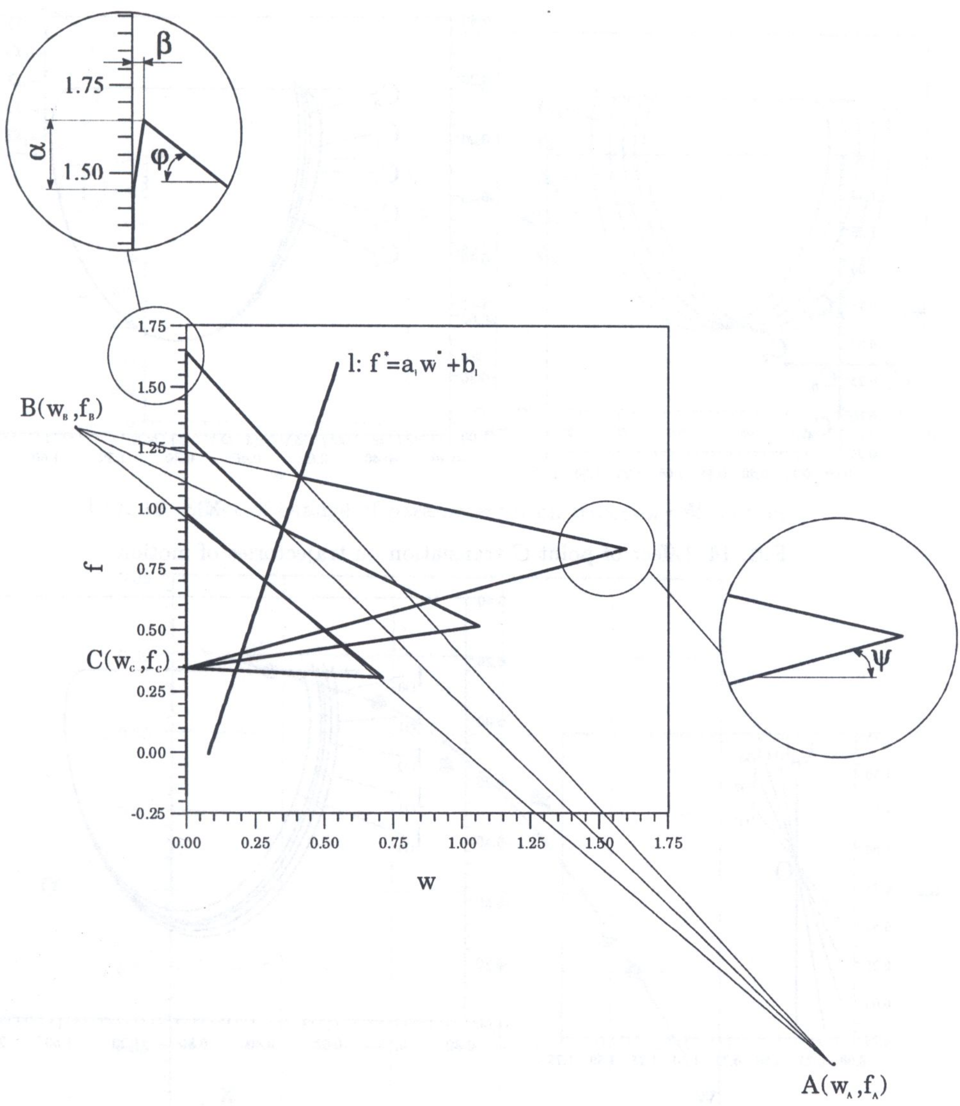

FIG. 13. Geometrical parameters of friction model.

It follows from the analysis, that one can obtain different shapes of phase trajectories by the change of friction model parameters. It is caused by the change of damping intensity of the system in various phases of the vibrations cycle.

As a result of identification of the model, the parameters have been determined as follows: $\alpha = \alpha(f_s)$ (see Fig. 17), $\beta = 0.01$, $\varphi = -58°$, $B(-0.41; 1.33)$, $\psi = 39°$.

Studying compatibility of the experimental and theoretical phase trajectories, one can notice discrepancies. They are caused both by the limited precision of

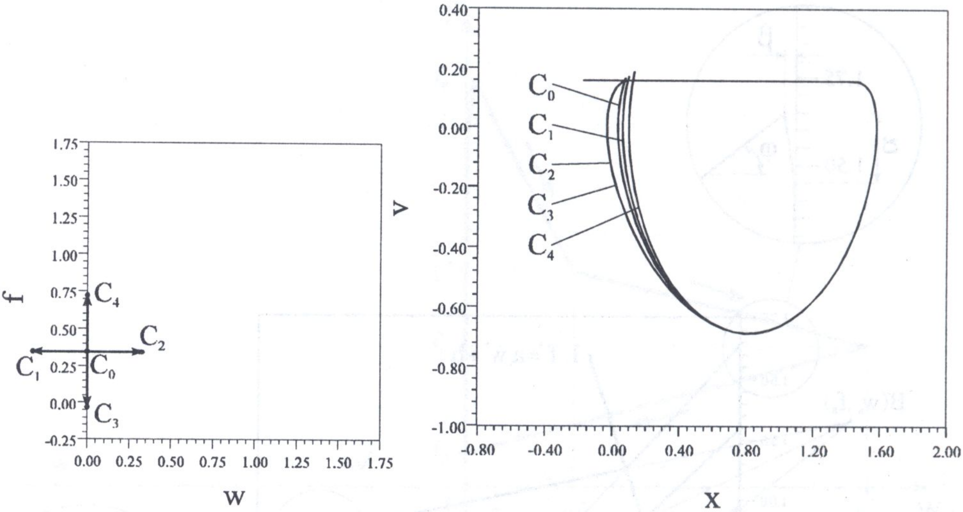

FIG. 14. Effect of point C translation on trajectories of motion.

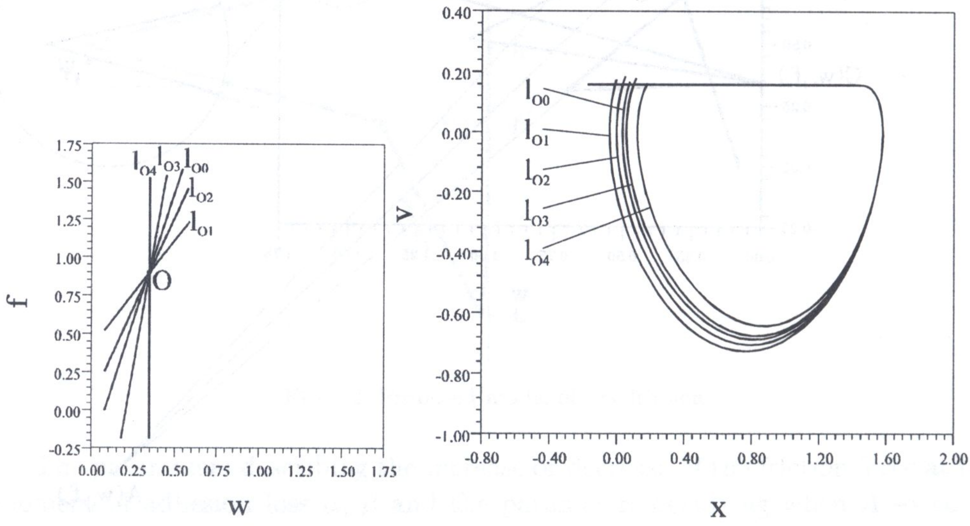

FIG. 15. Effect of rotation of straight line l around point O on trajectory of motion.

the measurement system and by the assumed simplification of the theoretical analysis. However, for an assumed friction model with characteristic independent of the sign of acceleration [5], much greater differences both, quantitative (fixed by values of displacements and velocities) and qualitative (referring to periodicity of the motion) in comparison to the proposed model can be seen. In Fig. 18 shapes of the phase trajectories experimentally obtained (marked by a broken line) and theoretically calculated in the case of the proposed friction model (a solid heavy line) as well as theoretically calculated trajectory in the case of the

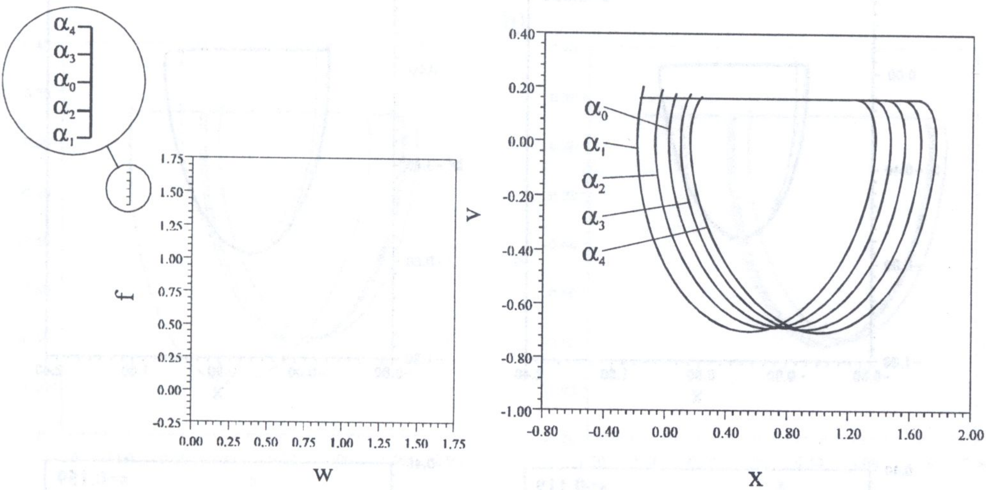

FIG. 16. Effect of change of parameters α on trajectory of motion.

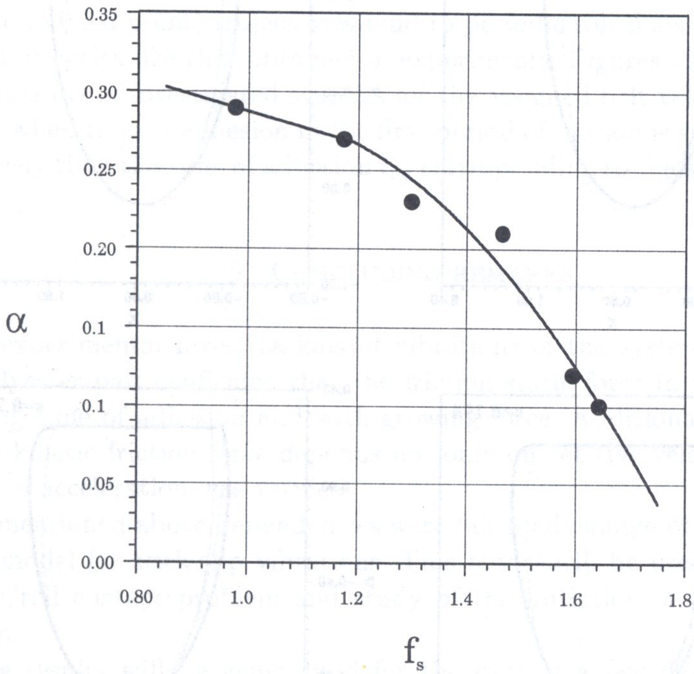

FIG. 17. Parameter α as a function of statical friction force f_s.

model independent of the sign of acceleration (a solid light line) for different velocities of the moving space are shown.

As a result of the computer simulation of the system motion for the proposed friction model one can observe, that for any initial conditions (except the condi-

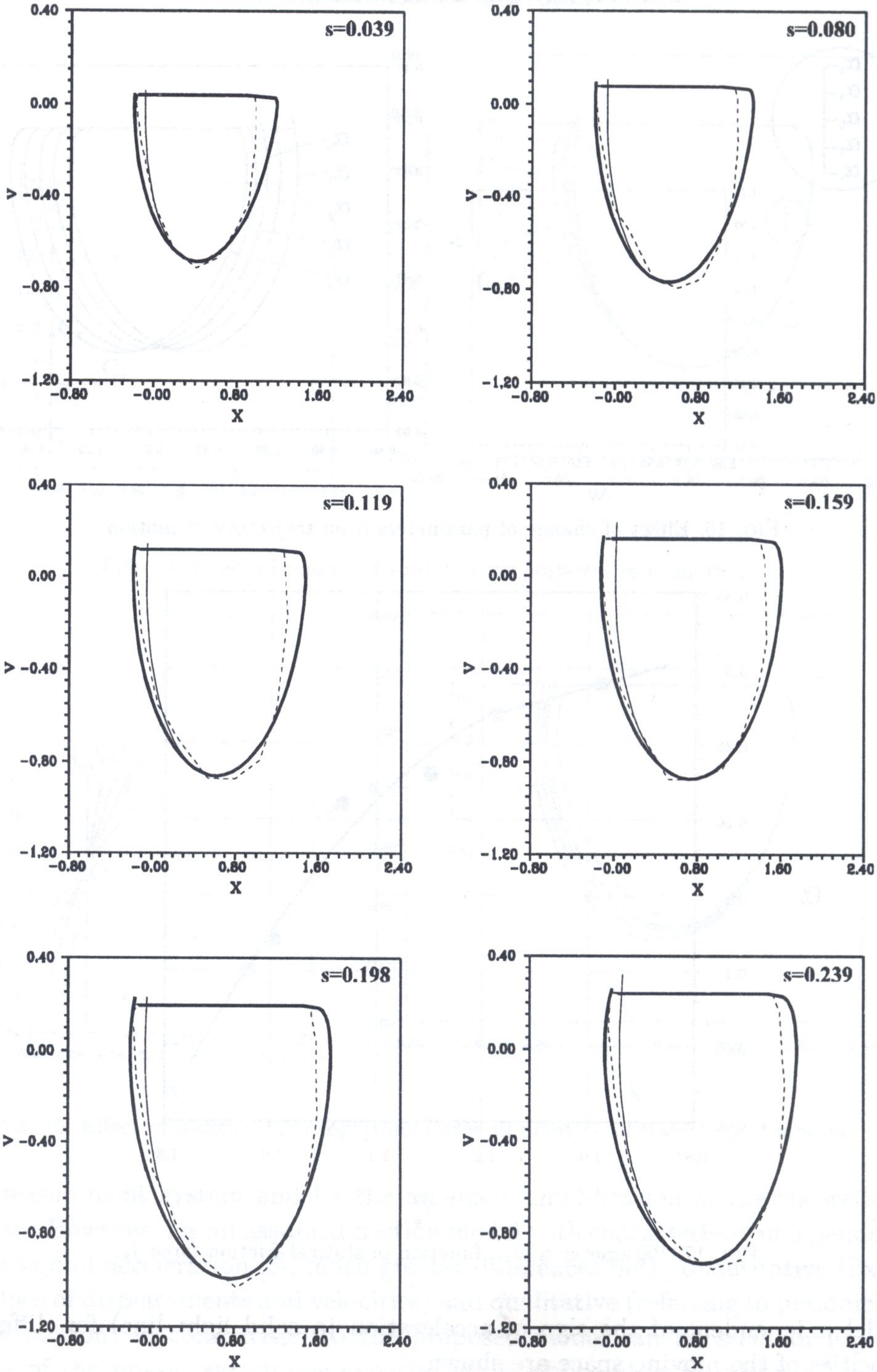

FIG. 18. Comparison of theoretically calculated to experimentally obtained phase trajectories.

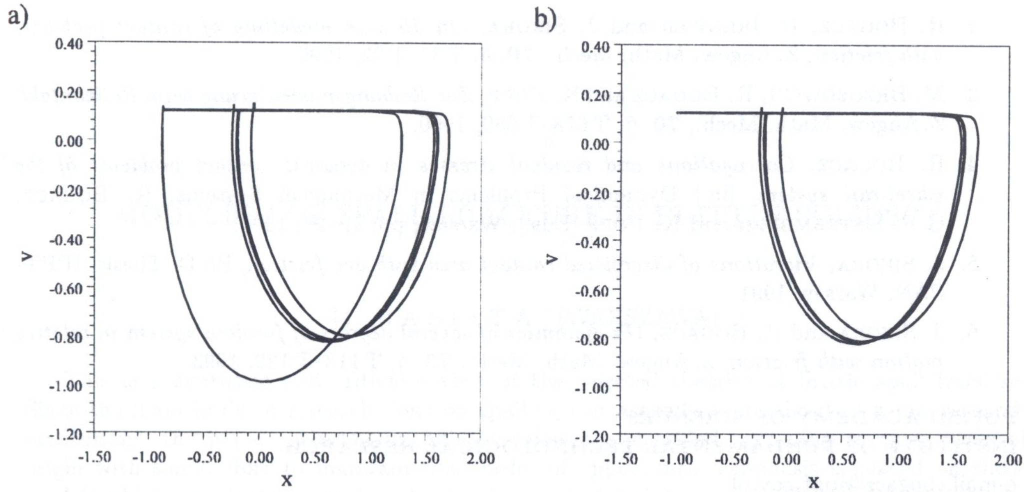

FIG. 19. Phase portraits for s = const in cases, when: a) $t_{s1} < t_{cg}$ and b) $t_{s1} > t_{cg}$.

tions due to fixed point) trajectories tend to periodic solutions corresponding to stable limit cycles like that obtained in experiments. Figures 19 a and 19 b show phase portraits of investigated systems for the assigned belt velocity (s = const) in cases, when time of adhesion in the first period of motion is smaller and larger respectively than the time of adhesion t_{cg} corresponding to the stable limit cycle.

5. Concluding remarks

The experimental investigations of vibrations of the system composed of a steel–polyester pair confirmed that the friction static force increases both with increasing time of adhesion and with growing force. Additionally, a statement, that the kinetic friction force depends not only on relative velocity but also on the sign of acceleration was verified.

The mentioned above dependencies were taken advantage of to formulate the friction model for stick-slip vibrations. This model will be used in the analysis of wheel/reil contact problem and study of the initiation of corrugation phenomenon.

These results will be generalised for the case of a few degrees of freedom systems.

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POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

e-mail:rbogacz@ippt.gov.pl

e-mail:bryczek@ippt.gov.pl

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