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LIMIT ANALYSIS OF STRUCTURES WITH DESTRUCTIBLE ELEMENTS UNDER IMPACT LOADINGS

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Limit states and identification of structures with shock- or seismic-protected system under dynamic loadings are discussed. Such structures include both the destructible (elastic-brittle) and indestructible (elastic-plastic) elements. A mathematical model and algorithm for solving shakedown problem of bearing capacity of systems with destructible elements are suggested. Next the propagation of vibrations from impacts of Minsk subway trains into nearby skeleton of 9-storied building is investigated. The experimental data for this building are received. Then the propagation of vibrations is analyzed numerically. Finally, a technique of minimax to evaluate dynamic elastic modules of concrete in the considered structure elements is used.

Key words: shock- or seismic-protected system, brittle and plastic elements, limit states, skeleton buildings, subway trains, parameter identification, experiment.

1. Introduction

Protection of buildings and structures from shock and seismic actions is very important. One way of protection is equipment of their load-bearing structures with systems of different elements absorbing the energy of external actions [1–3]. Some elements can be abruptly shut down (elastic-brittle ones), and some can be damaged as a result of plastic flow (elastic-plastic ones). This paper presents a mathematical model, proposed by the first of author, of the limit analysis of systems with destructive elements providing such protection. Next, by the example of 9-storey residential building located near the subway shallow in Minsk, the identification of its computational model is considered [4–11]. Significant sensitivity of stiffness of joints between reinforced concrete columns and floor slabs as a result of dynamic analysis of the building was shown.

The problem of protection of buildings from vibration caused by the movement of subway trains has also lately acquired specific urgency, when at construction of new lines of subway railway, the tunnels of shallow subway have been generally laid. This way of the tunnels laying has technical and economic advantages in comparison with the tunnels at the great depth, and comes to be the basic one at present. However, in the buildings located close to the tunnels of the shallow subway, vibrations achieve such a level that they become perceptible for people being inside [12–14]. To reduce the vibrations, it is necessary to study them in order to efficiently select or create new means of decreasing vibration level in residential and public buildings, being under construction or constructed close to the tunnels of shallow subway [15, 16]. Existing methods of vibration insulation, that use steel springs and vibration absorbers set between the foundation and upstream construction elements, do not take into account the peculiarities of vibration propagation within a building. These methods of the vibration insulation are basically used for the existing buildings.

The remoteness of buildings and constructions from sources of vibration corresponding to norm SNB 3.01.04–02 [17] should make 100 m for railway and 40 m for the shallow subway. In practice, according to [18], about 24% of areas defined above are built up. The organization of such areas is first of all caused by an attempt to reduce a harmful impact of vibration on people and buildings, not resorting to expensive methods of vibration decreasing.

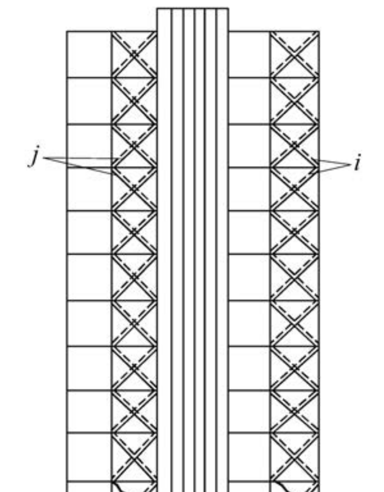

Fig. 1. Scheme of seismic protection of tall building by combined strengthening: i are brittle-destructible ties; j are elastic-plastic ties [19].

Protection of buildings against seismic and shock actions is achieved by equipping them with systems bearing structures of various elements absorbing energy of these actions. As a result, the destruction of the basic structure is prevented. Some elements can be abruptly shut down (elastic-brittle elements), and some damaged as a result of plastic flow (elastic-plastic elements). By the way of illustration these may be ties, guys, fascicles, strands, and other elements which are off in the process of shock or earthquakes.

Two examples of protection systems, for tall building and for nuclear power plant with a seismic isolation foundation and combined elements mounted on the frame, are shown in Figs. 1, 2. It was found [19] that after 5 cycles of plastic deformations and brittle-destructions of protected elements for 8- and 7-intensity scales, the seismic loads on the structures are reduced accordingly by 50% and 30%.

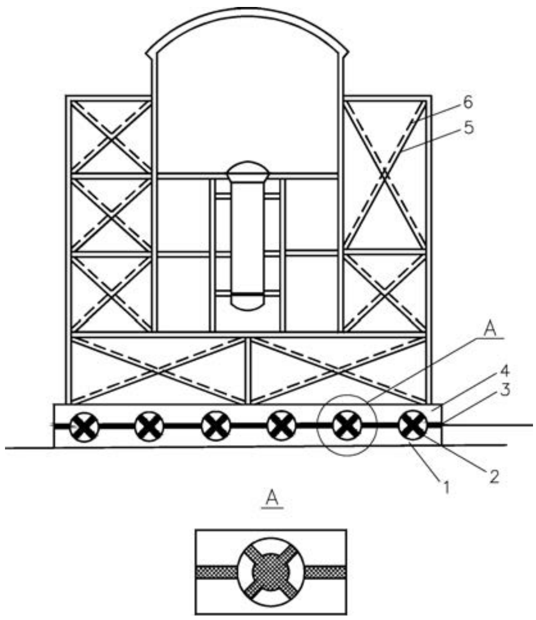

Fig. 2. Basic scheme of reactor with seismic safety foundation and combined strengthening elements on the building frame: 1 is lower foundation plate; 2 are damping supports; 3 is sliding support; 4 is upper foundation plate; 5 are elastic-plastic ties; 6 are brittle-destructible ties [19].

2. A mathematical model of structures with shock- or seismic-protected systems

Let us assume the problem of load-bearing capacity of such structures as a generalized dynamic shakedown problem [20–22], taking into account small system displacements under low cyclic external actions. First we write the FEM equation of motion for a damped discrete elastic system under loading $\mathbf{F}$, expressed in matrix notation, as follows:

$$[M]\,\ddot{\mathbf{u}} + [C]\,\dot{\mathbf{u}} + [K]\,\mathbf{u} = \mathbf{F}, \tag{2.1}$$

where $[M]$, $[C]$, $[K]$ are accordingly structural mass, damping and stiffness matrices; $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$, $\mathbf{u}$ are accordingly nodal acceleration, velocity and displacement vectors; $\mathbf{F}$ is a vector of load as a function of time t.

The vector $\mathbf{F}$ belongs to the set $\Omega_{\mathbf{F}}$, described by the vectors of single loadings $\mathbf{F}_j$, $j \in J$; J is a set of single loadings. The set $\Omega_{\mathbf{F}}$ has to include a natural structures stress state $\mathbf{F} = \mathbf{0}$ (i.e. $\mathbf{0} \in \Omega_{\mathbf{F}}$) and generally is nonconvex (grey domain in Fig. 3), [20]. For the purpose of simplification it may be approximated by the convex domain (bordered by firm line in Fig. 3).

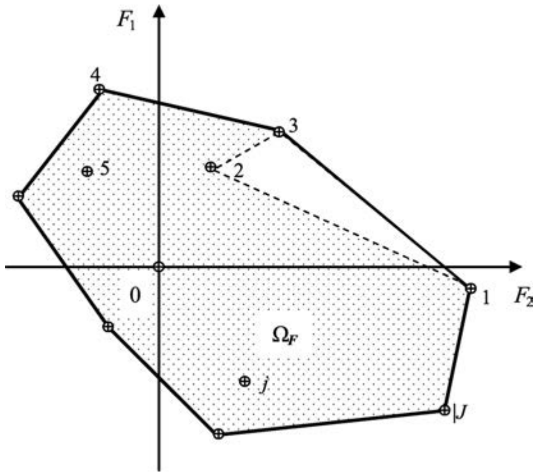

FIG. 3. The set $\Omega_{\mathbf{F}}$ of system loading $\mathbf{F}$, described by the vectors of single loadings $\mathbf{F}_j$, $j \in J$, for $\mathbf{F} \in \mathbf{R}^2$.

The "elastic" solution of Eq. (2.1) is used then as a basis for analysis of real inelastic system. Namely the problem of load-bearing capacity of structures made of perfectly elastic-plastic and elastic-brittle elements, under variable loads, is formulated as follows. Find the vectors of single loadings $\mathbf{F}_j$, $j \in J$, a vector of load $\mathbf{F}$, as well as the vector of residual forces $\mathbf{S}^r = (\mathbf{S}^r_{pl},\ \mathbf{S}^r_{br})$ such, that

$$\sum_{j \in J} \mathbf{T}^T_{\mathbf{F}j}\mathbf{F}_j \to \max, \tag{2.2}$$

$$\boldsymbol{\varphi}_{pl/\Omega\mathbf{F}}(\mathbf{S}^e_{pl} + \mathbf{S}^r_{pl}, \mathbf{K}_{pl}) \le \mathbf{0}, \tag{2.3}$$

$$\boldsymbol{\varphi}_{br}(\mathbf{S}^e_{br} + \mathbf{S}^r_{br}, \mathbf{K}_{br}) \le \mathbf{0}, \tag{2.4}$$

$$\mathbf{S}^e = \boldsymbol{\omega}_{\mathbf{F}}\mathbf{F}, \tag{2.5}$$

$$\mathbf{A}_{pl}\mathbf{S}^r_{pl} + \mathbf{A}_{br}\mathbf{S}^r_{br} = \mathbf{0}, \tag{2.6}$$

$$\mathbf{S}^r_{br} \ge \mathbf{0}_{br}. \tag{2.7}$$

Here $\mathbf{T}_{\mathbf{F}j}$ are the vectors of weight coefficients corresponding to the vectors of single j-loading $\mathbf{F}_j$, $j \in J$; $\boldsymbol{\varphi}_{pl/\Omega\mathbf{F}}$ are the yield functions, depending on the set $\Omega_{\mathbf{F}}$ external actions (loadings $\mathbf{F}_j$) for elastic-plastic elements; $\boldsymbol{\varphi}_{br}$ are the strength functions for brittle elements; $\boldsymbol{\omega}_{\mathbf{F}}$ is the matrix of loads influence on the elastic forces $\mathbf{S}^e = (\mathbf{S}^e_{pl},\ \mathbf{S}^e_{br})$; $\mathbf{A} = (\mathbf{A}_{pl},\ \mathbf{A}_{br})$ is a matrix of equilibrium Eqs. (2.6). The subscripts *pl* and *br* relate to the elastic-plastic and elastic-brittle elements, superscripts e and r – to the elastic and residual forces.

After finding the failure mechanism (active constraints (2.4)) in problem (2.1)–(2.7) one must take into account the dynamic effects of this destruction in iterative procedure [23]. The simple approach to such dynamic analysis was proposed in [24].

Note that the formulated above problem (2.1)–(2.7) for mixed structures with elastic-plastic and elastic-brittle elements is new. In addition to loads $\mathbf{F}_j$, in this problem it is possible to consider the dislocations $\mathbf{d}_j$, $j \in J$, as similar external actions. By changing the dislocation $\mathbf{d}_j$ we can also optimize the state of structures prestressing.

In the particular case of one-pass loading, the problem (2.1)–(2.7) is simplified when $|J| = 1$, this problem is also applicable for the analysis of the progressive collapse of structures [25].

3. Description of the research building nearby the subway tunnels

Investigation of vibration propagation within the building structure caused by the movement of the rolling stock of the subway, have been executed on a residential building being under construction, which is located in the Pritytskiego street in Minsk. This building has five sections (parts), with different number of floors in each section. The first section is completed and has 9 floors. The third section that has 9 floors is at the initial stage of construction: the foundation and 3 floors (at the moment of carrying out of measurements) have been completed. The analysis of parameters of vibration has been executed for the fourth 9-storied section, that is the highest and the most closely located to the subway (∼28 m up to the axis of a tunnel).

Construction works have not been completed at the moment of carrying out the measurements. Works on construction of coating on the 9th floor and the garret floor were performed, what has resulted in a raised level of a vibration background at the moment of carrying out the measurements. The plan of the building divided into sections and the axis location of the subway tunnel line relatively to the building are presented in the Fig. 4.

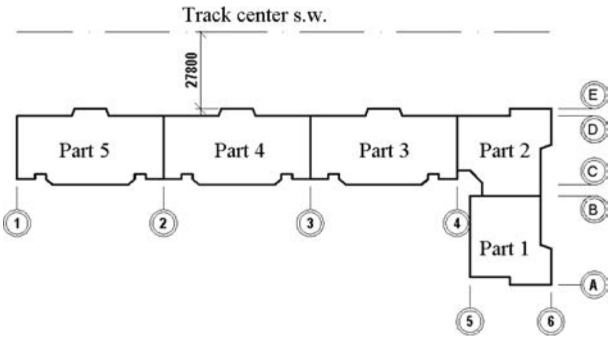

Fig. 4. The plan of the building and location of the subway line axis.

The given section represents monolithic skeleton system with longitudinal and cross-section shear wall, and supported by floor walls made of cell concrete

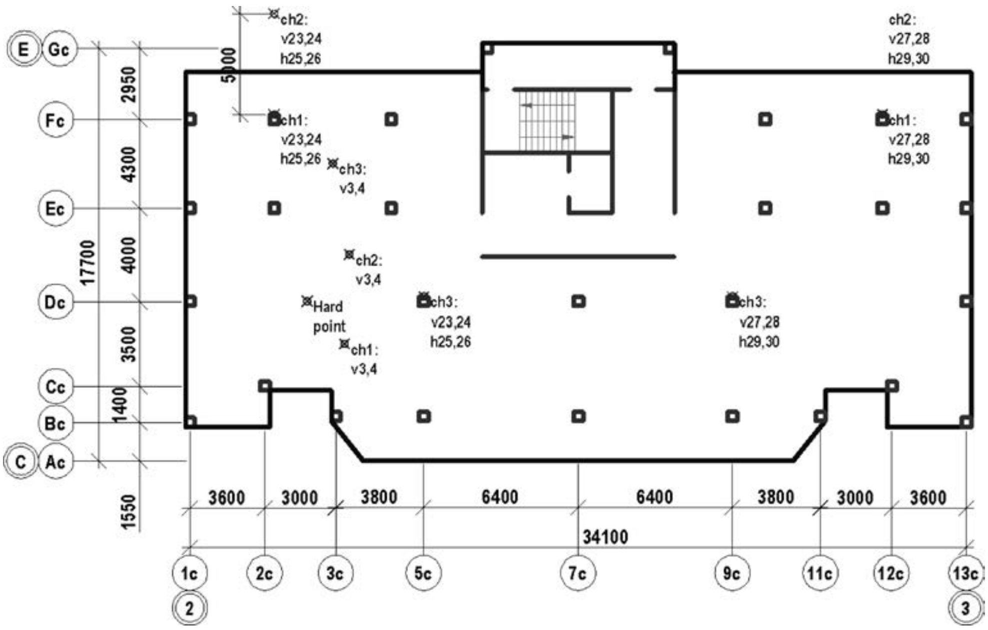

Fig. 5. The constructive scheme of a typical floor of the fourth section of the 9-floor residential unit.

blocks. Floor slabs are monolithic, 200 mm thick, with concealed beam heads. The basement part along the perimeter of the building is made of massive concrete, 400 mm thick. Columns cross-section size is 400×400 mm. The constructive scheme of a typical floor of the fourth section of the 9-storied monolithic-frame residential unit is presented in the Fig. 5.

4. Measurements of vibration for the soil and for the construction foundation

The measurement of vibrations was performed by means of a four-channel measuring system with use of a piezoceramic accelerometer. At passing of the subway trains, the greatest changes of the spectrum were observed in the range of frequencies from 1 to 100 Hz [9]. Measurements of vibration have been taken on the soil in front (∼3.5 m) of the building and in the basement, as well as on the construction elements of the basement part of the building: the shear wall and the columns. Due to the busy schedule of construction works, the total measurements within the whole building were not possible at the moment of measuring. The general layout of measurement points in the basement part of the building is presented in the Fig. 5.

The results of iterative measurements of vibration (not less than 3 hours per each point, an interval of trains movement is about 5 min) are shown in Figs. 6–8. Envelope curve of the narrow-band acceleration spectrum is denoted

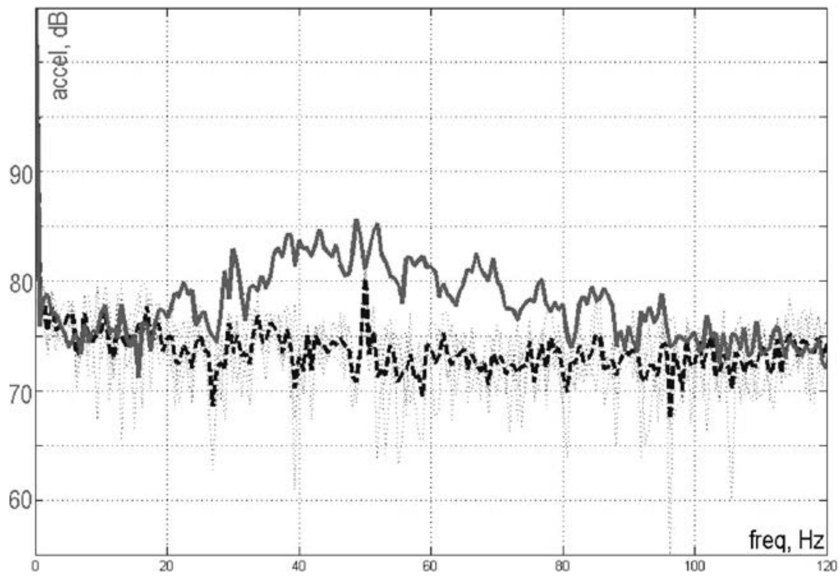

Fig. 6. The spectrogram of vertical levels of vibration of the soil in the point ch1-v23.

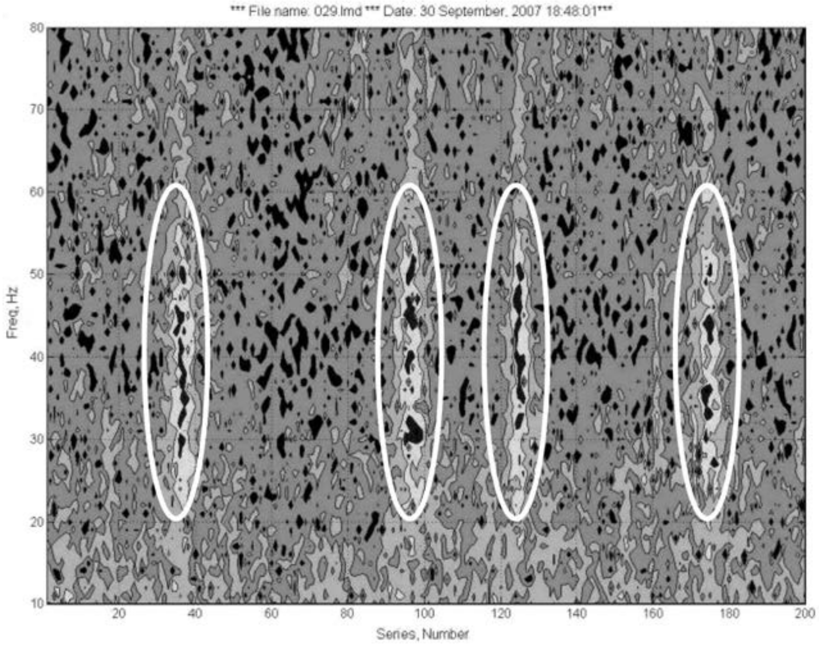

FIG. 7. The spectrogram of horizontal levels of vibration of the soil in the point ch2-h29.

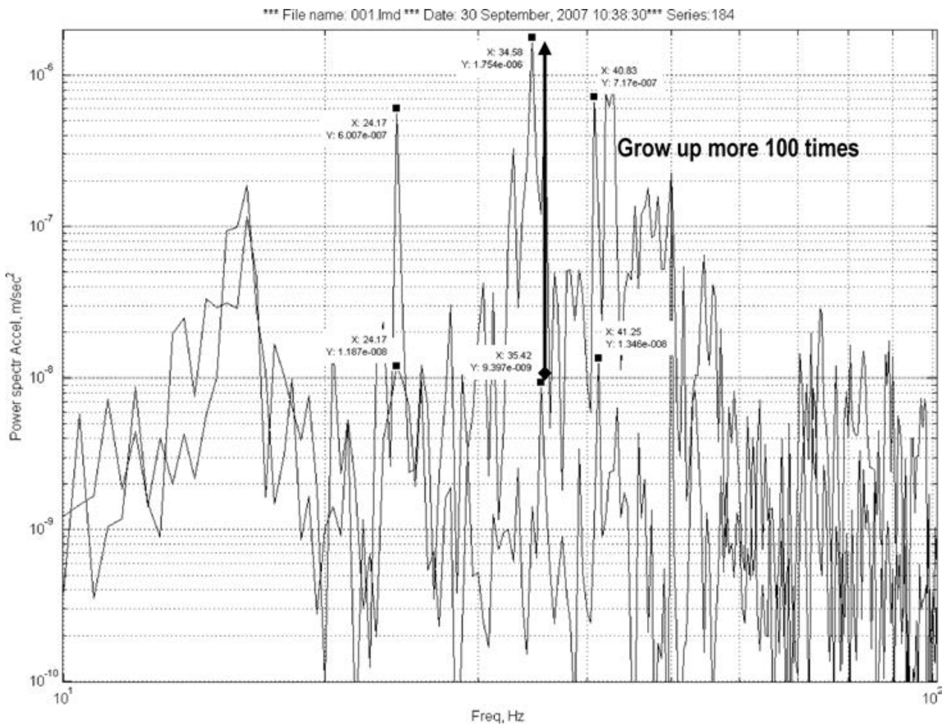

FIG. 8. The spectrogram of vertical levels of vibration of a column in the point ch3-v23.

by a solid line, levels of the background are denoted by a dashed line. The levels L_a of acceleration in these figures,

$$L_a = 20 \cdot \mathrm{Log}_{10}\left(\frac{a}{1 \cdot 10^{-6}}\right), \tag{4.1}$$

are measured in dB; a is acceleration (ref 1E-6), m/s^2.

At passing of subway trains it has been revealed that with the increase in amount of passengers (the rush hour is between 17 and 19 o'clock), the levels of acceleration is 3...3.5 dB higher if compared with the period between 19 and 22 o'clock. Besides, the levels of acceleration depend on the technical state of the railway, the rolling stock, and on the speed of the stock movement (for the rush hours, as a rule, the speed is higher) [13, 26]. All the factors listed above affect the frequency distribution of the levels of acceleration. The peaks of vibration in the range of frequencies of 20 to 60 Hz are displaced in relation to some central frequency (Fig. 8).

At low frequencies, in the range of 1 to 10 Hz, the changes of the levels of acceleration are insignificant in relation to the background and therefore have not been further analyzed.

As a definition of objectively accurate borders of spectral bands for the forced vibration is not obviously possible without total modal analysis, further calculation is performed by means of replacing a real chart of distribution of vibration levels from frequency by a two-level chart (Fig. 9) with a level of 0.001 m/s^2 (60 dB) for the range of frequencies from 1 to 100 Hz and 0.03 m/s^2 (90 dB) for the range of frequencies from 20 to 60 Hz. The calculation is executed by means of the spectral method on a random vibration.

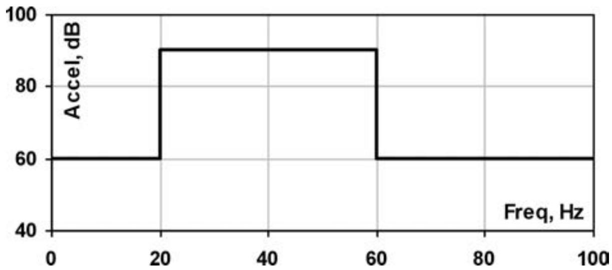

FIG. 9. The spectrum of acceleration for the building calculation on a random vibration.

5. Modelling of vibrations

The modal FEM analysis is used for natural frequencies and mode shapes determination. The Eq. (2.1) of motion for a damped system is as following:

$$[M]\,\ddot{\mathbf{u}} + [C]\,\dot{\mathbf{u}} + [K]\,\mathbf{u} = \mathbf{0}. \tag{5.1}$$

The scheme of the finite elements net of the investigated building fragment is presented in Fig. 10. The shell finite elements of the SHELL63 type are taken here (in the FEA ANSYS) for the slabs and stiffening diaphragms, and bar finite elements BEAM4 for the columns. The total number of elements is 6528.

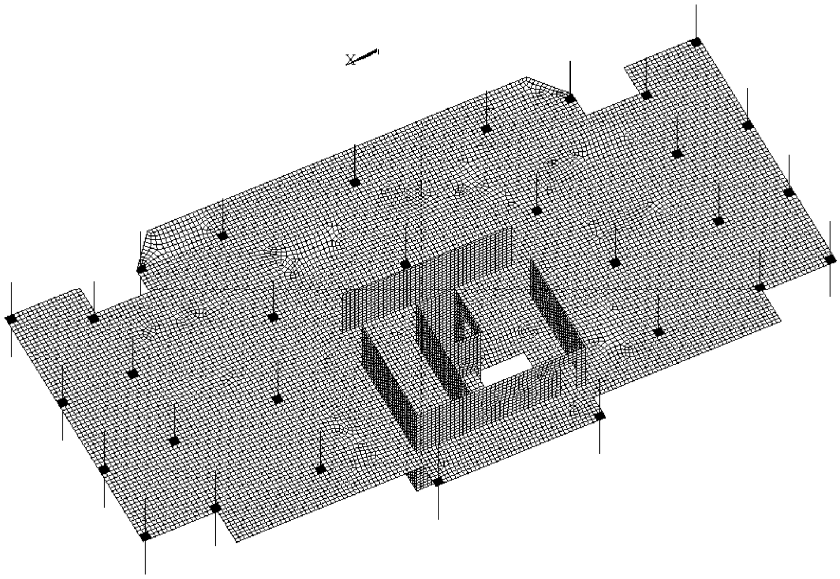

FIG. 10. Design model of the investigated building fragment.

For a linear system without damping, natural vibrations will be harmonic of the form:

$$\mathbf{u} = \boldsymbol{\phi}_i \cos\ \omega_i t, \tag{5.2}$$

where $\boldsymbol{\phi}_i$ is the eigenvector representing the mode shape of the i-th natural frequency; ω_i is the i-th natural circular frequency (radians per unit time); t is time.

After substitution Eq. (5.2) into (5.1), the eigenvalue and eigenvector problem has the form:

$$[K]\boldsymbol{\phi}_i = \lambda_i [M]\boldsymbol{\phi}_i, \tag{5.3}$$

where λ_i is i-th eigenvalue.

This problem is solved here by the block Lanczos eigenvalue extraction method. For each eigenvector $\boldsymbol{\phi}_i$, normalization with the mass matrix is:

$$\boldsymbol{\phi}_i^T [M]\boldsymbol{\phi}_i = 1. \tag{5.4}$$

Then the spectrum analysis is used for the nondeterministic, random vibration method with excitations at the support. In its part, the random vibration method is based on the power spectral density (PSD) approach. The damping ratio is equal to 0.05. The displacement, velocity or acceleration vector for each mode is computed from the corresponding eigenvector taking into account a "mode coefficient":

$$\mathbf{r}_i = \omega_i^2 A_i \boldsymbol{\phi}_i, \tag{5.5}$$

where A_i is spectral acceleration for the i-th mode.

For excitation acceleration of the base we have:

$$A_i = \frac{S_{ai}\gamma_i}{\omega_i^2}, \tag{5.6}$$

where S_{ai} is spectral acceleration for the i-th mode (obtained from the input acceleration response spectrum at frequency $f_i = \omega_i/(2\pi)$ and effective damping ratio ξ_i').

The participation factors, for the given excitation direction (i-th mode), are defined as:

$$\gamma_i = \boldsymbol{\phi}_i[M]\mathbf{D}, \tag{5.7}$$

where $\boldsymbol{\phi}_i$ is eigenvector normalized using Eq. (5.4); $\mathbf{D}$ is a vector describing the excitation direction.

The vector $\mathbf{D}$ has the form:

$$\mathbf{D} = [T]\mathbf{e}, \tag{5.8}$$

where

$$\mathbf{D} = [\,\mathbf{D}_1^a \quad \mathbf{D}_2^a \quad \mathbf{D}_3^a \quad \ldots]^T, \tag{5.9}$$

$\mathbf{D}_j^a$ is excitation at the j-th Degree of Freedom (DOF) in the direction a; a may be either any axis X, Y, Z or rotation about one of these axes;

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & (Z-Z_0) & -(Y-Y_0) \\ 0 & 1 & 0 & -(Z-Z_0) & 0 & (X-X_0) \\ 0 & 0 & 1 & (Y-Y_0) & -(X-X_0) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

X, Y, Z are global Cartesian coordinates of a point on the structure space; X_0, Y_0, Z_0 are global Cartesian coordinates of a point about which rotations are done (reference point); $\mathbf{e}$ denotes six possible unit vectors.

For spectrum analysis, the values $\mathbf{D}^a$ may be determined as:

$$\mathbf{D}_X = \frac{S_X}{B}, \qquad \mathbf{D}_Y = \frac{S_Y}{B}, \qquad \mathbf{D}_Z = \frac{S_Z}{B}, \tag{5.10}$$

where S_X, S_Y, S_Z are components of excitation direction; $B = \sqrt{S_X^2 + S_Y^2 + S_Z^2}$.

The modal displacements, velocity and acceleration are combined by complete quadratic combination method to obtain the response of the structure

$$R_a = \left(\left| \sum_{i=1}^{N} \sum_{j=1}^{N} k\varepsilon_{ij} R_i R_j \right| \right)^{1/2}, \tag{5.11}$$

where

$$k = \begin{cases} 1 & \text{if } \; i = j, \\ 2 & \text{if } \; i \neq j, \end{cases} \qquad \varepsilon_{ij} = \frac{8\sqrt{\xi_i'\xi_j'}\left(\xi_i' + r\xi_j'\right) r^{3/2}}{(1-r^2)^2 + 4\xi_i'\xi_j'(1+r^2) + 4(\xi_i'^2 + \xi_j'^2)r^2},$$

$r = \omega_j/\omega_i$, $\qquad \xi_{i,j}'$ is effective damping ratio for the mode i, j.

The combined value "Sum",

$$\text{Sum}_i = \sqrt{X_i^2 + Y_i^2 + Z_i^2}, \tag{5.12}$$

is calculated for one node of X, Y and Z displacement.

6. Numerical procedure of parameter identification

Identification of the constitutive parameters holds true for three material constants (modules of concrete elasticity), accordingly E_{p1} for slab, E_{p3} for joint slab with column and E_c for column (Fig. 10), which combine into a vector of parameters of system $\mathbf{x} = (E_{p1}, E_{p3}, E_c) \in \mathbf{R}^n$, $n = 3$.

Criterion of a minimax identification of system in terms of [27, 28],

$$\rho(\mathbf{x}) = \max_{i \in I} \left| \frac{f_{mi} - f_{ci}(\mathbf{x})}{(f_i^+ + f_i^-)/2} \right|, \tag{6.1}$$

has to minimize under Eq. (5.1), and known initial conditions, where f_{mi} are measured values of i-th natural frequencies, $f_i = \omega_i/(2\pi)$, $i \in I$; I is a set of analyzed frequencies; $f_{ci} \equiv f_{ci}(\mathbf{x})$ are values of i-th natural frequencies, calculated from Eq. (5.1) and known initial conditions, depending on the vector $\mathbf{x}$; f_i^+, f_i^- are accordingly high and low values of i-th natural frequencies of vibration given by the designer. Three natural frequencies were investigated here, $|I| = 3$.

As far as the identification of system is an ill-posed problem, criterion of optimality (6.1) is modified

$$\rho_\alpha(\mathbf{x}) = \rho(\mathbf{x}) + \alpha \left\|\mathbf{x}\right\|^2, \tag{6.2}$$

where α is a Tikhonov regularization parameter [29], $\alpha > 0$; $\|\mathbf{x}\| = \sqrt{\sum\limits_{i\in 1:n} (x_i^2)}$.

Choice of regularization parameter α on k-step of calculations process, $k = 0, 1, \dots K$, was made by the formula

$$\alpha_k = \alpha_0 q^k, \qquad q > 0. \tag{6.3}$$

The next parameter α_{k+1} is found from the following $(K+1)$-th problem:

$$\left\|\mathbf{x}_{\alpha_{k+1}} - \mathbf{x}_{\alpha_k}\right\| \to \min_{k\in 0:K}, \tag{6.4}$$

up to the comprehensible accuracy of solving an initial problem (6.2).

The received optimum vector $\mathbf{x}^*$ of parameters of the investigated 9-storied building contains the following values of modules of concrete elasticity: for the slabs $E_{p1} = 30.24$ GPa (reduction of the reference modulus by 19.05%) and for the columns $E_c = 47.54$ GPa (increment the reference modulus for 18.85%). Change of modulus of concrete elasticity for the joints of slabs with columns E_{p3} comes to 5.3 times. Comparison of natural frequencies of the fragment vibration, found by reference with the results of measurements, is shown in Table 1.

Table 1. The optimum parameters of building constructions received by decision of the identification problem for the investigated fragment.

The parameter name	Reference values	Optimum values	The measured values
Modulus of concrete elasticity, Pa			
E_{p1} (slab)	3.6E+10	3.024E+10	–
E_c (column)	4.0E+10	4.754E+10	–
E_{p3} (slab in region joint with column)	3.6E+10	1.911E+11	–
Natural frequencies of vibration for investigated fragment, Hz			
FREQS04	17.877	16.601	16.25
FREQS13	26.554	23.428	24.17
FREQS24	35.748	34.582	35.00
Parameters of system optimality on Eqs. (6.1)–(6.4)			
TST	0.136	0.397E-01	–
TST1	0.91E-2	0.456E-03	–
TST2	0.88E-2	0.972E-03	–
TST3	0.45E-3	0.144E-03	–

The difference between reference, optimum and measured values of natural frequencies is small, but it leads to the distinction in kind of vibration forms, as is shown in Fig. 13 for the frequencies of 40.682 Hz and 40.721 Hz accordingly.

7. Analysis of constructure vibration

A typical feature of the vibration of construction elements for the four sections of the 9-storied building model for the range of frequencies from 0 to 10 Hz at calculation for a random vibration (the propagation of the basis by a random force is set by means of a spectrum of acceleration of bearers), is a prevalence of the horizontal component of displacement above the vertical one. Natural frequencies of construction vibrations for the range of frequencies from 0 to 10 Hz are presented in the Table 2.

Table 2. Natural frequencies of vibrations for the range from 0 to 10 Hz.

No	1	2	3	4	5	6
Frequency, Hz	1.72	2.38	3.23	5.84	9.07	9.73

It is necessary to note that at low frequencies, the greatest contribution to the process of the building vibration is introduced by the horizontal component along the axis Y-Y or X-X, but with the increase in frequency, the greatest contribution to the vibration process is introduced by the vertical component of vibration, along the axis Z-Z.

The distributions of the vibration accelerations (Sum_a, see Eq. (5.12)) over construction elements of the model for the ranges of frequencies accordingly 0–20, 20–40, 40–60 Hz by calculation for a random vibration, are presented in Figs. 11a-c. The distribution of vibration accelerations in the vertical plane Z-Z over construction elements of the model, for the range of frequencies from 0 to 20 Hz by calculation for a random vibration, is presented in the Fig. 12.

The numbers of forms of normal vibration frequencies for the specified foregoing ranges are presented in the Table 3.

Table 3. Natural vibrations frequencies of the model of the building.

Frequency, Hz	0–20	20–40	40–60	60–80
Number of eigenvalues	186	278	287	278

The analysis of normal frequencies and forms of vibration in the range of frequencies from 1 to 60 Hz shows that components of horizontal displacement prevail over the vertical ones for the lowest frequencies between 1 and 7 Hz.

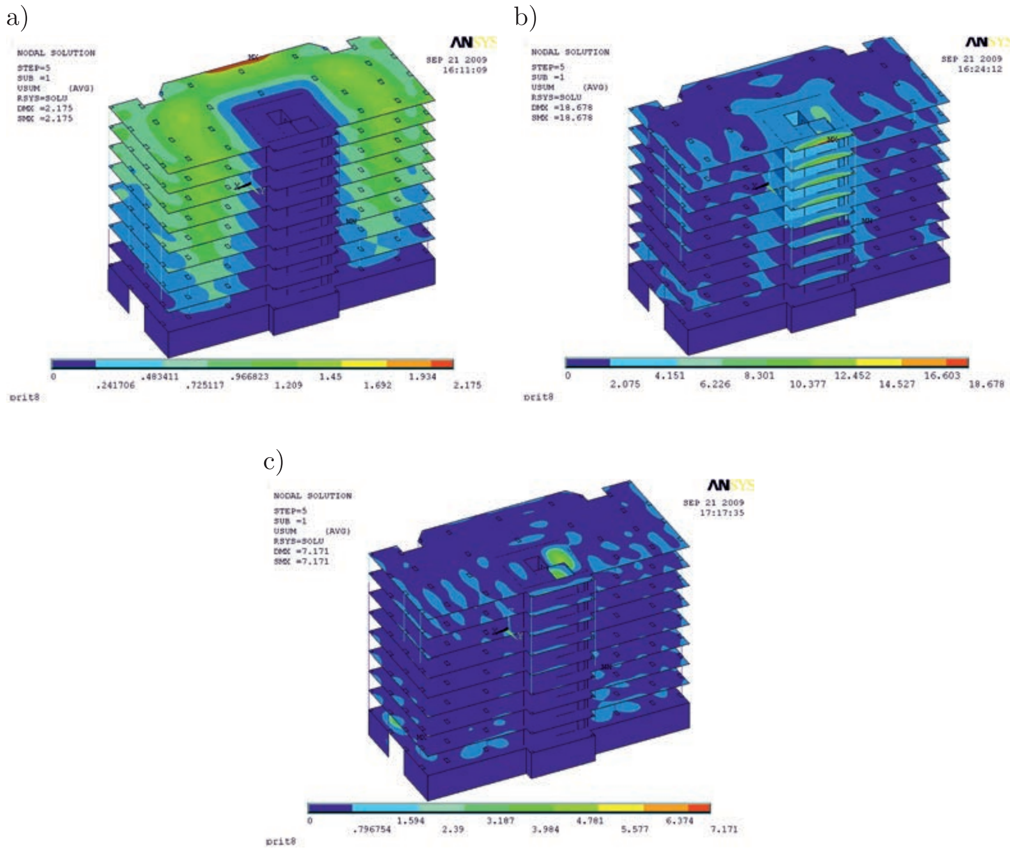

FIG. 11. Vibration accelerations (Sum) of construction elements by calculation for random vibration: a) for the range of frequencies, from 0 to 20 Hz; b) the same, from 20 to 40 Hz; c) the same, from 40 to 60 Hz.

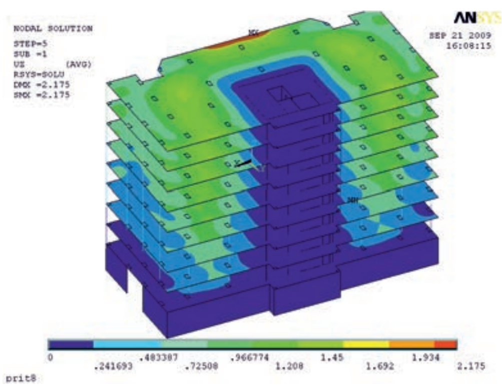

FIG. 12. Vibration accelerations of construction elements (Z-Z) by calculation for random vibration for the range of frequencies, from 0 to 20 Hz.

With the increase in frequency, the vertical component of vibration becomes a prevailing component in the vibrating mode. Therefore, the vibration analysis of the rolling stock of the subway is considered mainly in a plane of floor disks for 1–9 floors.

The proportion of the maximal amplitudes for the displacements of construction elements (Sum:Z:Y:X, related to displacement Sum) for different forms of normal vibration in the range of frequencies from 1 to 60 Hz are presented in the Table 4. The maximal amplitudes of displacement specified in this table along X, Y and Z axes are presented for various nodes of the model within the limits of one form of vibration; however the value "Sum" is defined for the node with the maximal displacement defined by the Eq. (5.12).

Table 4. Proportion of the maximal amplitudes for the displacements along the orthogonal axes *X*, *Y*, *Z*.

Frequency, Hz	Sum	Z	Y	X
1.72	1.00	0.13	0.21	0.98
2.38	1.00	0.10	0.79	0.48
3.23	1.00	0.31	0.85	0.72
5.84	1.00	0.33	0.60	0.84
9.07	1.00	0.34	0.50	0.90
9.73	1.00	0.99	0.64	0.75
11.02	1.00	1.00	0.12	0.08
20.13	1.00	1.00	0.67	0.02
35.01	1.00	1.00	0.29	0.10

A building being under construction gives a possibility of free planning. As a result, there can appear areas with significant levels of vibrations in case the cross-walls are irrationally placed. Occurrence of low-frequency noise in empty accommodations (the effect of rumbling) is also possible. It is determined that areas with excessive vibration can be formed on any floors of a building, from the 1st to the 9th. At various forms of normal vibrations of construction elements, the areas stated above can be present on different floors. For example, for the frequency of 40.682 Hz, the maximal displacement of the floor is formed on the 9th floor, and for the frequency of 40.721 Hz is on the 3rd floor (Fig. 13).

The calculation has shown that occurrence of such areas is possible for any range of frequencies, even in the areas limited by rigid elements (low deformable elements in the vertical plane relatively to the bending of the floor): staircase, shear wall, column.

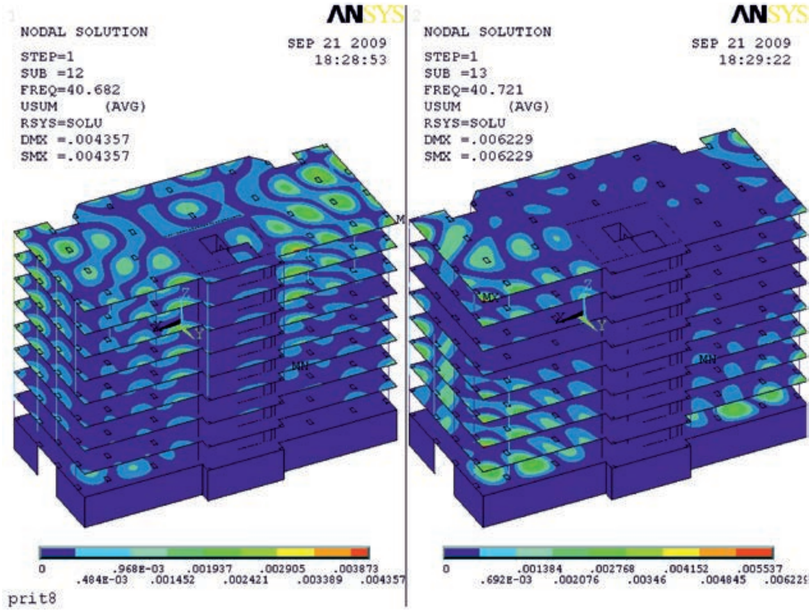

FIG. 13. Location of areas with the maximal vibration accelerations on different floors, corresponding to different forms of vibration.

The vibration of construction elements for the same nodes, located on the vertical line between the 1st and 9th floors and caused by random acceleration of the foundation from the external impact with a set function of spectral density for various frequency ranges, has been investigated. Results of calculation are presented in the Fig. 14.

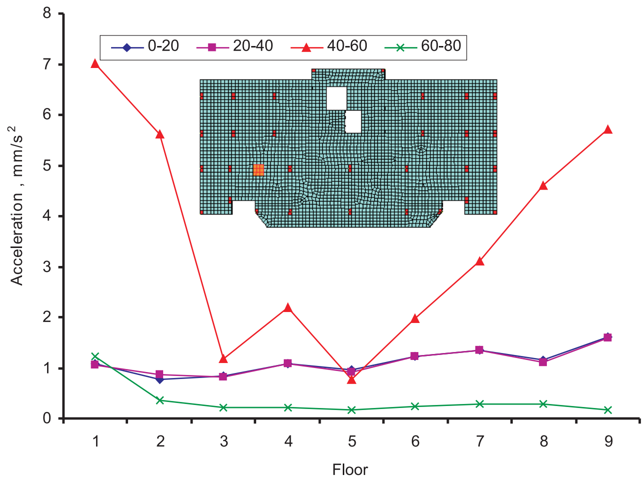

FIG. 14. The graph of change of the acceleration amplitude by the floors in the nodes, coinciding vertically.

The graphs of maximal amplitude of constructions acceleration for the floor disk and shear wall, calculated for the different ranges of frequencies, are presented in Figs. 15 and 16 accordingly. The levels of vibration essentially differ only for the range of frequencies from 20 to 60 Hz; for the range of frequencies from 0 to 20 Hz, the acceleration of the floor disks between 1st and 9th stories changes insignificantly and, on the whole, is presented by a horizontal line.

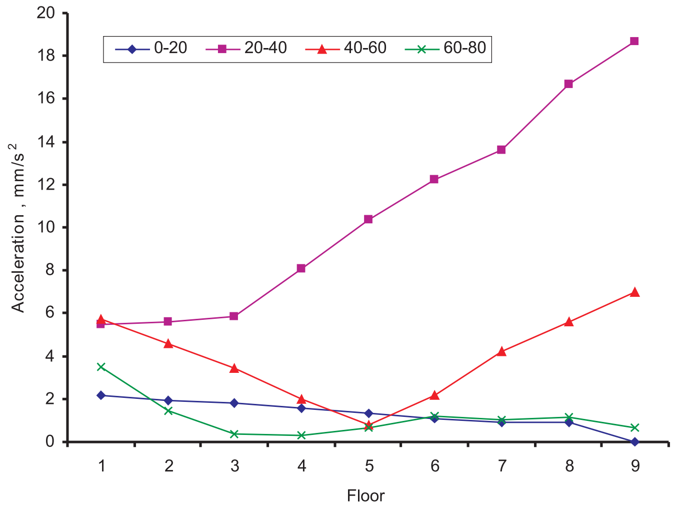

FIG. 15. The graph of change of acceleration amplitude by the floors in nodes with the maximal amplitude of acceleration.

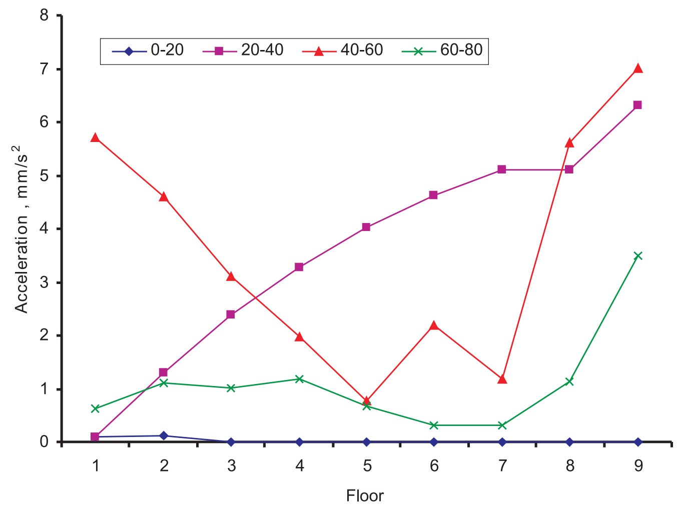

FIG. 16. The graph of change of acceleration amplitude by the shear wall in nodes with the maximal amplitude of acceleration.

8. Conclusions

A mathematical model and algorithm for solving of the problems of bearing capacity of shock- or seismic-protected systems with destructible elements are suggested.

A technique of minimax to evaluate dynamic elastic modules of concrete for the skeleton 9-storied building, which is located nearby Minsk subway tunnel, was used.

The greatest acceleration obtained for the shear wall of the investigated 9-storied building corresponds to the range of frequencies of 20–60 Hz, practically for all floors. The highest levels of acceleration of the floor disk are obtained for the 1st, 8th and 9th floors, and therefore at measurements, modal analysis and development of constructive actions, it is advisable to perform a more complete investigation on distribution of vibration over the construction elements in the above-mentioned range of frequencies.

The changes of model parameters as a result of identification for concrete elasticity modules are equal to about 20% for the slabs and columns, whereas for the joints of slabs with columns this change comes to more than 5 times. Consequently, the initial design model of the structure has to be modified, for example, by installing rigid inclusions in the joints of slabs and columns.

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