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MODELLING THE PLASTIC FLOW OF COMPOSITE MATERIALS IN THE EXTRUSION PROCESS

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The paper deals with an attempt of modelling the plastic flow of composed material based on the experimental results of co-extrusion of various materials. A modified mathematical description of the plastic flow in the extrusion of the bimetallic composite has been proposed. Boundaries of the plastic zone are described by appropriate functions adapted according to special choice of the proposed parameters. The flow field, the grid distortion and strain rate distribution have been approximated taking into account information on the plastic zone boundaries and their form. Different modes of simultaneous plastic deformation of dissimilar metals which are generally different from a mono-metal deformation are presented. Excellent agreement between the analytical and the experimental results of the plastic flow of composite has been found.

1. Introduction

Progress in the production of metallic composites by extrusion depends on suitable designing of such types of material and its plastic deformation. Development of the analytical methods requires a good description of multi-material plastic flow. Nevertheless, even the deformation process in the extrusion of bimetallic rods composed of dissimilar metals remains unclear and needs deformation characteristics peculiar to the bimetallic composite reflected by the suitable velocity field.

Up to now, most of the authors (e.g. Avitzur [1, 2]) have said, that "sound co-extrusion" is possible when different materials deformed simultaneously represent proportional flow only. But different structures and properties of various materials deformed together make such mechanical behaviour impossible. This fact was reflected in some papers (e.g. [3, 4]) dealing with experimental analysis of such type of composed flow and [5] with an analytical point of view.

The results of the latter work allowed their use in the upper bound approach. Finally, they enable us to obtain velocities, strain rate tensor and stresses with accuracy to an unknown hydrostatic pressure. Different points of view are represented in the papers [6, 7, 8]. The obtained stress fields are completely defined under some strong additional assumptions concerning the model of plastic flow of composite material, or by the FEM method.

This paper presents the results of mathematical modelling of the plastic flow of composed material based on the detailed experimental results of co-extrusion of different materials.

The flow field, the grid distortion and the strain distribution have been approximated taking into account information on plastic zone boundaries and its form. They are compared with the corresponding experimental results. At the first stage, the mechanical parameters of simultaneously deformed materials are not included directly in the solution, but they are reflected by the proposed geometrical parameters of plastic zone, which are dependent on the mechanical features of the components of the composite and their volume ratio. Proper parameters describing the plastic zone can be expected to provide a better prediction of the material flow as well as the load accepted to the upper-bound method. In this study, mathematical modelling of the composite plastic flow is based on the analysis of plastic zone in co-extrusion (as a result of different mechanical and geometrical features of the composed material to be deformed). It has been developed on the basis of observations of actual composite material flow. The description of the plastic zone is presented using an appropriate function adopted according to the special selection of the proposed parameters. Here, a modified mathematical description of plastic flow of composed material able to reflect actual material flow in the extrusion of a bimetallic composite has been proposed. Different modes of simultaneous deformation of dissimilar metals which are generally different from a mono-metal deformation are presented.

In the paper [5], a kinematically admissible velocity field of the bimaterial extrusion has been proposed and discussed to analyze such type of the extrusion process. In that case, the boundaries of the plastic zone are approximated by ellipsoidal, hyperboloidal and sinusoidal surfaces. But the results of other papers (e.g. [3, 9]) show that plastic zone may be bounded by arbitrary shape of the surfaces.

The objective of this work is:

- to propose a description of general form of plastic zone boundaries with some special assumptions,
- to estimate internal consistency of the proposed kinematically admissible velocity field,
- to compare the analytical and new experimental results concerning velocities, strain rate and stress fields obtained in [3, 4, 10].

2. Problem formulation and simplifying assumptions

Basing on the analysis of the flow patterns of the actual materials deformed together and the results of the previous papers (e.g. [5]), it is reasonable to accept different geometry of the deformation boundaries between various materials deformed together (the core and the sleeve) and between the entry and exit surfaces of each of the materials.

Taking this into account, additional assumptions are applied:

1. Relative velocity (V/V_0) is the same for every component of the composite at the entry of the plastic zone. Hence, there is no friction between the materials. Besides, it is assumed that there is no friction along the material-tool interface.

2. Both materials of the core and the sleeve demonstrate plastic regions ($ABDC$ and $ABFE$, respectively see Fig. 1). Geometry of the deformation boundaries is different in the core and the sleeve, as well as in the entry and exit boundaries of each material. The plastic regions depend essentially on the mechanical properties of the materials and friction along the metal-metal and metal-tool interfaces. However, such dependence is very complicated and should be established due to the upper bound method.

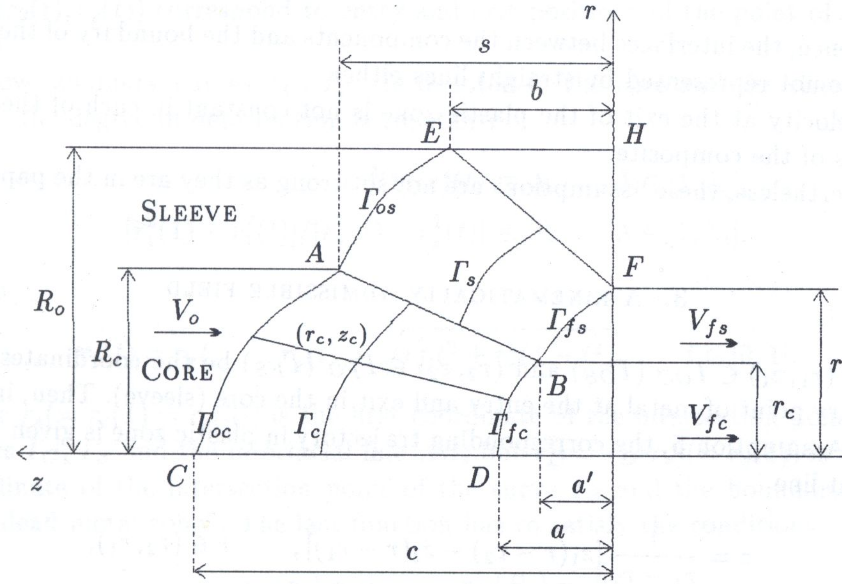


Fig. 1.

3. During co-extrusion of different materials dead metal zone may exist. Its form depends on the geometry of extrusion tools and the properties of materials extruded together. For general consideration it is essential to examine formation of the dead zone during extrusion using flat dies.

4. The materials are assumed to be incompressible and the stress-strain state is prescribed by the rigid-plastic model.

5. Flow lines in the plastic zone are straight lines for both the sleeve and core materials.

6. The degree of deformation is different for the core and the sleeve:

$$\lambda_c = R_c^2/r_c^2\,, \qquad \lambda_s = (R_o^2 - R_c^2)/(r_o^2 - r_c^2), \qquad \lambda_c \neq \lambda_s \neq \lambda_{\text{global}}\,. \tag{2.1}$$

However, it is similar in each of the material components, so the exit velocities of components are "constant" but not the same.

Such assumptions enable us to simplify a model of the plastic flow, which is similar to that proposed in [5]. Of course, such model is an approximation of the real plastic flow, and has certain disadvantages. The following points could be noted:

- Mechanical parameters are introduced in an indirect way only.
- The velocity field is discontinuous along the entry and exit boundaries of the plastic zone.
- Trajectory of an arbitrary point in the plastic zone is not a straight line, in general.
- Hence, the interfaces between the components and the boundary of the dead zone are not represented by straight lines either.
- Velocity at the exit of the plastic zone is not constant in each of the components of the composite.

Nevertheless, these assumptions are not so strong as they are in the paper [6].

3. A kinematically admissible field

Let $(r_1, z_1) \in \Gamma_{OC}$ (Γ_{OS}) and $(r_2, z_2) \in \Gamma_{FC}$ (Γ_{FS}) be the coordinates of an arbitrary point of metal at the entry and exit in the core (sleeve). Then, in view of the Assumption **5**, the corresponding trajectory in plastic zone is given by the straight line

$$z = \frac{1}{r_1 - r_2}\left[z_1(r - r_2) - z_2(r - r_1)\right], \qquad r \in (r_2, r_1), \tag{3.1}$$

where (r, z) is the successive position of the point.

It is assumed that the boundaries of the plastic zone are prescribed by the following curves determined by their parametric forms:

$$\Gamma_{OC}\ :\ \begin{cases} z_1 = f_1^c(t), \\ r_1 = R_c t, \end{cases} \qquad \Gamma_{FC}\ :\ \begin{cases} z_2 = f_2^c(t), \\ r_2 = r_c t, \end{cases} \qquad t \in [0,1], \tag{3.2}$$

in the core, and

$$(3.3)\qquad \Gamma_{OS}: \begin{cases} z_1 = f_1^s(t), \\ r_1 = \sqrt{R_o^2 t^2 + R_c^2(1-t^2)}, \end{cases} \qquad \Gamma_{FS}: \begin{cases} z_2 = f_2^s(t), \\ r_2 = \sqrt{r_o^2 t^2 + r_c^2(1-t^2)}, \end{cases} \qquad t \in [0,1],$$

in the sleeve.

Such description of the plastic zone boundaries guarantees that Assumption **5** is satisfied, and an arbitrary form of these curves can be chosen by determining the functions f_j^c, f_j^s $(j = 1,2)$. The last one must only satisfy additional conditions dealing with the geometry of the plastic zone (see Fig. 1):

$$(3.4)\qquad \begin{aligned} &f_1^c(0) = c, \quad f_1^c(1) = s, \quad f_2^c(0) = a, \quad f_2^c(1) = a', \\ &f_1^s(0) = s, \quad f_1^s(1) = b, \quad f_2^s(0) = a', \quad f_2^s(1) = 0. \end{aligned}$$

Besides, note that for each value of parameter $t \in (0,1)$ the points $(r_1(t), z_1(t))$ and $(r_2(t), z_2(t))$ correspond to entry and exit positions of the point of the material.

Now, auxiliary curves Γ_C, Γ_S are denoted in the core and the sleeve along which the degree of deformation is constant:

$$(3.5)\qquad \begin{aligned} r_1^2(t)/r_c^2(t) &\equiv \lambda, \qquad \lambda \in [1, \lambda_c], \\ [r_1^2(1) - r_1^2(t)]/[r_s^2(1) - r_s^2(t)] &\equiv \lambda, \qquad \lambda \in [1, \lambda_s], \end{aligned}$$

hence

$$(3.6)\qquad r_c(t) = r_* t, \qquad r_s(t) = \sqrt{r_D^2 t^2 + r_*^2(1-t^2)}, \qquad t \in [0,1],$$

where $r_* (= r_c(1) = r_s(0))$ is the first coordinate of the intersection point of the curves Γ_C, Γ_S and the interfacial line AB, but $r_D = r_D(r_*)(= r_s(1))$ is the first coordinate of the intersection point of the curve Γ_S and the boundary EF of "the dead metal zone". The last function has to satisfy the conditions

$$r_D(r_c) = r_o\,, \qquad r_D(R_c) = R_o\,.$$

Precise form of $r_D(r_*)$ is not known *a priori*. For example, it can be described as follows:

$$(3.7)\qquad r_D(r_*) = \left(\frac{R_o^\alpha - r_o^\alpha}{R_c^\alpha - r_c^\alpha}(r_*^\alpha - r_c^\alpha) + r_o^\alpha \right)^{1/\alpha}, \qquad \forall \alpha \in (0, \infty).$$

We shall assume, that

$$(3.8)\qquad r_D(r_*) = \sqrt{\frac{R_o^2 - r_o^2}{R_c^2 - r_c^2}(r_*^2 - r_c^2) + r_o^2}\,.$$

Such choice (3.7) (or (3.8)) can be additionally stipulated by the following argument. Namely, if both materials of the extrusion process are similar, then it is natural to assume that there is a proportional flow in the core as well as in the sleeve. Hence, the reduction parameters λ_c and λ_s (extrusion ratios) for an arbitrary point on the interfacial line are equal ($\lambda_c = \lambda_s$). It will be possible (see (3.1)) only, if

$$r_D = r_* \frac{R_o}{R_c}.$$

The last relation is equivalent to (3.7) in view of the assumption of proportionality of flow: $R_o/R_c = r_o/r_c$.

Finally, note that the second components of the points belong to the curves Γ_C, Γ_S, and are defined by the relation (3.1) with $r = r_c$ or $r = r_s$, respectively.

Let ϕ be the angle between axis OZ and the trajectory of each particle of metal defined by relation (3.1), but ϕ_N is the angle between the normal to the curve Γ_C in the core (Γ_S – in the sleeve) and the OZ-axis. Note that $\phi = \phi(t)$, $\phi_N = \phi_N(t, r_*)$, are defined from the relations:

$$(3.9)\qquad \begin{aligned} &\operatorname{tg}\phi = \frac{r_1(t) - r_2(t)}{z_1(t) - z_2(t)}, \qquad t \in (0,1),\\ &\operatorname{tg}\phi_N^c = \frac{dz_c}{dt}\frac{dt}{dr_c}, \qquad \operatorname{tg}\phi_N^s = \frac{dz_s}{dt}\frac{dt}{dr_s}, \qquad t \in (0,1), \quad r_* \in (r_c, R_c). \end{aligned}$$

Following the paper [5], due to incompressibility of the materials we can obtain:

$$(3.10)\qquad V(t, r_*) = \lambda_{c(s)}(r_*) V_0 \frac{\cos\phi(t)}{\cos[\phi(t) - \phi_N(t, r_*)]}, \qquad t \in (0,1), \quad r_* \in (r_c, R_c).$$

The above equation is applicable to both proportional and non-proportional flows. The axial and radial components are

$$(3.11)\qquad V_r = -V\sin\phi, \qquad V_z = -V\cos\phi.$$

Now, we can calculate strain the rate tensor,

$$(3.12)\qquad \dot\varepsilon_r = \frac{\partial v_r}{\partial r}, \qquad \dot\varepsilon_z = \frac{\partial v_z}{\partial z}, \qquad \dot\varepsilon_\theta = \frac{v_r}{r}, \qquad \dot\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right),$$

and finally, by the Saint-Venant - Levy - Mises hypotheses (see for example [11]), the stress tensor $\sigma'_{ij} = \sigma_{ij} - \sigma\delta_{ij}$ with accuracy to an unknown hydrostatic pressure σ is of the form:

$$\sigma'_{ij} = \frac{\sqrt{2}k}{\sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}}\dot{\varepsilon}_{ij}, \tag{3.13}$$

where $k = Y_c$ or $k = Y_s$ – yield stresses of the components of the composed material in the core and the sleeve, respectively.

4. Numerical results and discussion

First of all, we note some points which play an important role in the numerical realization of the model.

We shall calculate strain rate field basing on the relations presented above, and by using the parameters of the plastic zone (see Fig. 1) from the experimental results obtained in the papers [3, 4, 10]. At this time, the parameters R_o, R_c, r_o, r_c are calculated with a sufficient accuracy (with relative error less than 1%), but the remaining ones (a, a', b, s, c) can be found from the analysis of the kind of grid distortion and changes of macrostructure (see Fig. 2).

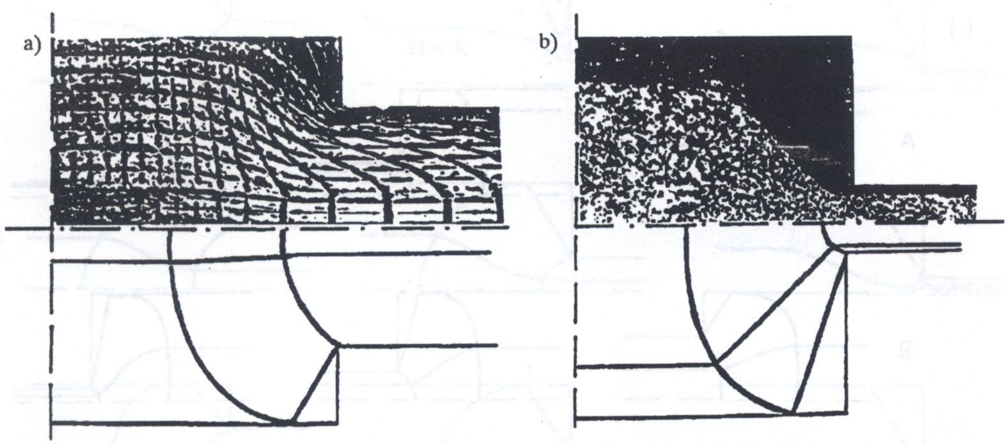


FIG. 2. a) Grid distortion of lead/hard lead alloy composite (hard core, soft sleeve) of type C geometry of the billet; $\lambda = 3$; b) macrostructure of the composite (soft core, hard sleeve) of type A geometry of the billet; $\lambda = 12$.

In the presented model, the following analytical forms of the functions

$$f(t) = A(t^{\beta} + B)^{\gamma}, \qquad \text{or} \qquad f(t) = A + B\sin^{\beta} Ct$$

are used. The proper choice of additional parameters defined the curvatures of the curves (see Figs. 3, 4) and allowed us to take into account friction on the metal-metal and metal-tool interfaces.

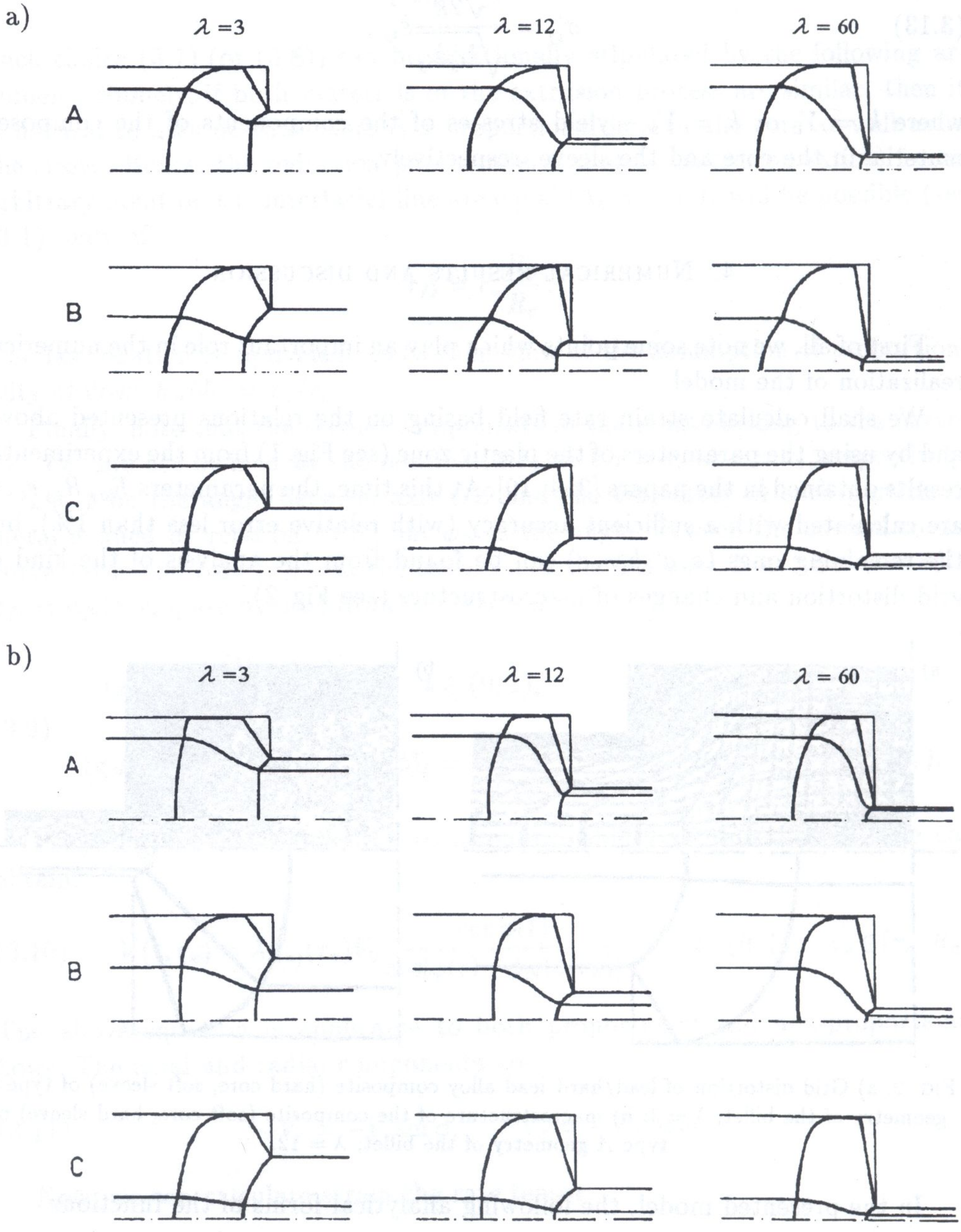


FIG. 3. Dependence of the shape of plastic zone of lead/hard lead alloy composite on billet geometry and extrusion ratio: a) hard core, soft sleeve; b) soft core, hard sleeve.

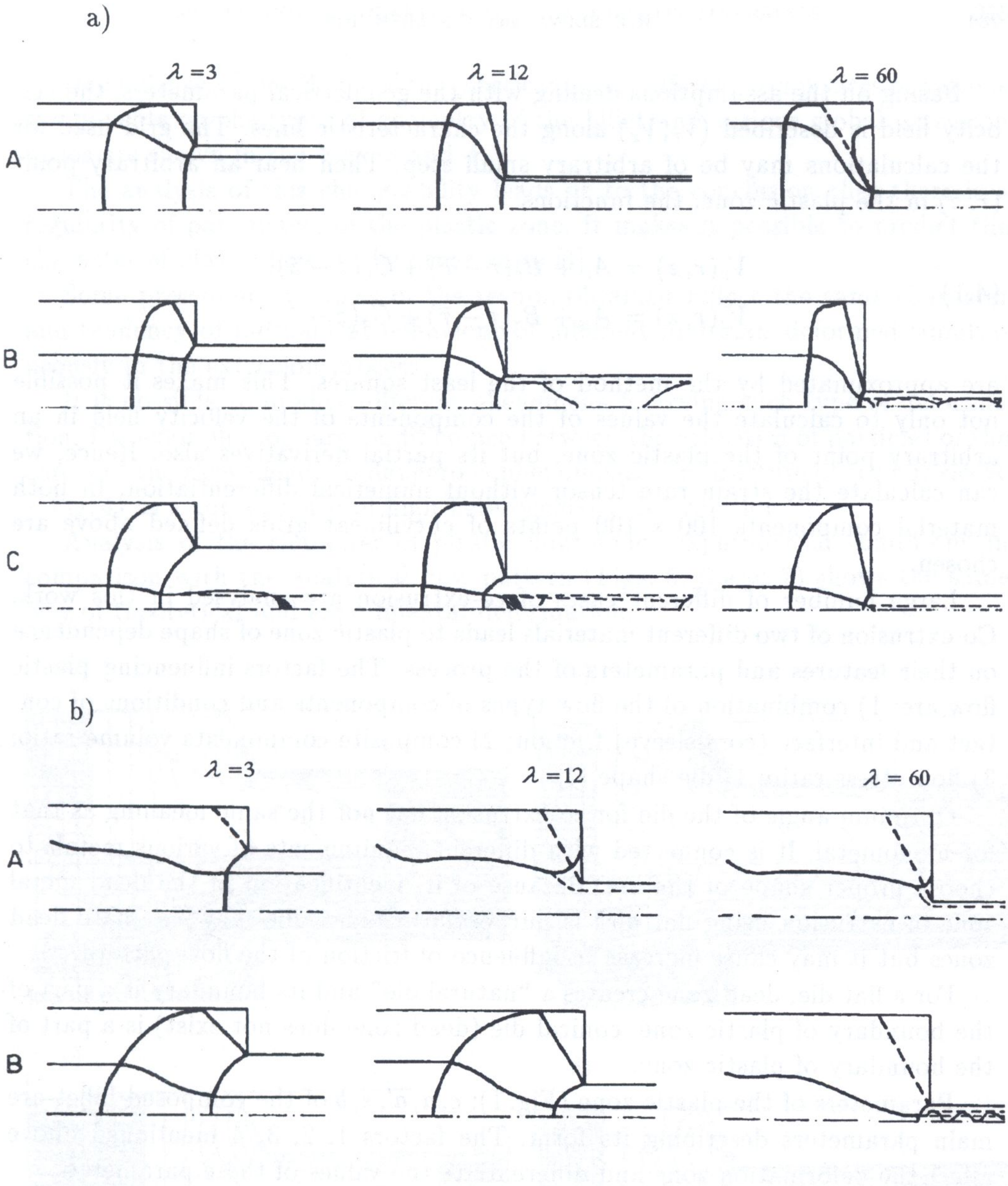


FIG. 4. Dependence of the shape of plastic zone ob Al/Pb composite on billet geometry and extrusion ratio: a) hard core, soft sleeve; b) soft core, hard sleeve.

Basing on the assumptions dealing with the geometrical parameters, the velocity field is described (V_r, V_z) along the characteristic lines. The grid used for the calculations may be of arbitrary small step. Then near an arbitrary point $(\overline{r}, \overline{z})$ in the plastic zone, the functions

$$\begin{aligned} V_r(r,z) &= A_r + B_r(r-\overline{r}) + C_r(z-\overline{z}), \\ V_z(r,z) &= A_z + B_z(r-\overline{r}) + C_z(z-\overline{z}), \end{aligned} \tag{4.1}$$

are approximated by the method of the least squares. This makes it possible not only to calculate the values of the components of the velocity field in an arbitrary point of the plastic zone, but its partial derivatives also. Hence, we can calculate the strain rate tensor without numerical differentiation. In both material components 100×100 points of curvilinear grids defined above are chosen.

Large number of different cases of co-extrusion are analysed in this work. Co-extrusion of two different materials leads to plastic zone of shape dependence on their features and parameters of the process. The factors influencing plastic flow are: 1) combination of the flow types of components and conditions of contact and interface (core-sleeve) friction; 2) composite components volume ratio; 3) flow stress ratio; 4) die shape [4].

Optimum angle of the die for co-extrusion has not the same meaning as that for monometal. It is connected with different requirements of various metals to choose proper shape of the die. Because of it, identification of the dead metal zone in extrusion using flat dies is purposeful. Conical die may eliminate dead zones but it may cause increase in influence of friction of the flow pattern.

For a flat die, dead zone creates a "natural die" and its boundary is a part of the boundary of plastic zone, conical die (dead zone does not exist) is a part of the boundary of plastic zone.

Parameters of the plastic zone (Fig. 1): c, a, a', s, b of the composed billet are main parameters describing its form. The factors 1, 2, 3, 4 mentioned above affect the deformation zone and differentiate the values of these parameters.

Identification of the plastic zone basing on grid distortion and macrostructure of the billet and the extruded product allows us to verify theoretical assumptions for modelling the process (e.g. Fig. 2).

The presented kind of modelling of the plastic flow of co-extrusion lets us obtain the grid distortion, velocity field, deviator of stresses and the strain rate tensor. Analysis of such behaviour basing on different materials (e.g. Al, duralumin, copper, lead, hard lead alloy) in various combinations and geometrical parameters [3, 4, 10, 12] gives a possibility to develop a theoretical method of description of plastic flow.

Dependence of the shape of the plastic zone of different composites of various components on the types of geometry of the billet and various global extrusion ratios are shown in the Figs. 3 and 4.

The analysis of this changeability leads us to the conclusion that there is a regularity of parameters of the plastic zone. It makes it possible to predict the character of plastic flow in the other cases [4].

Some presented examples of the results obtained reflect the same character and tendency of mechanical behaviour of different materials deformed simultaneously in the extrusion process.

It is possible to predict different phenomena following such type of deformation. For example, too large a difference between the velocities of particles of the core and the sleeve leads to dangerous phenomena (e.g. fracture of core or sleeve – Fig. 4) as a consequence of high heterogeneous deformation.

Analysis of the character of plastic flow under experimental conditions in comparison with the analytical flow pattern (Figs. 5, 6 and 7) shows the same way of theoretical and experimental deformation.

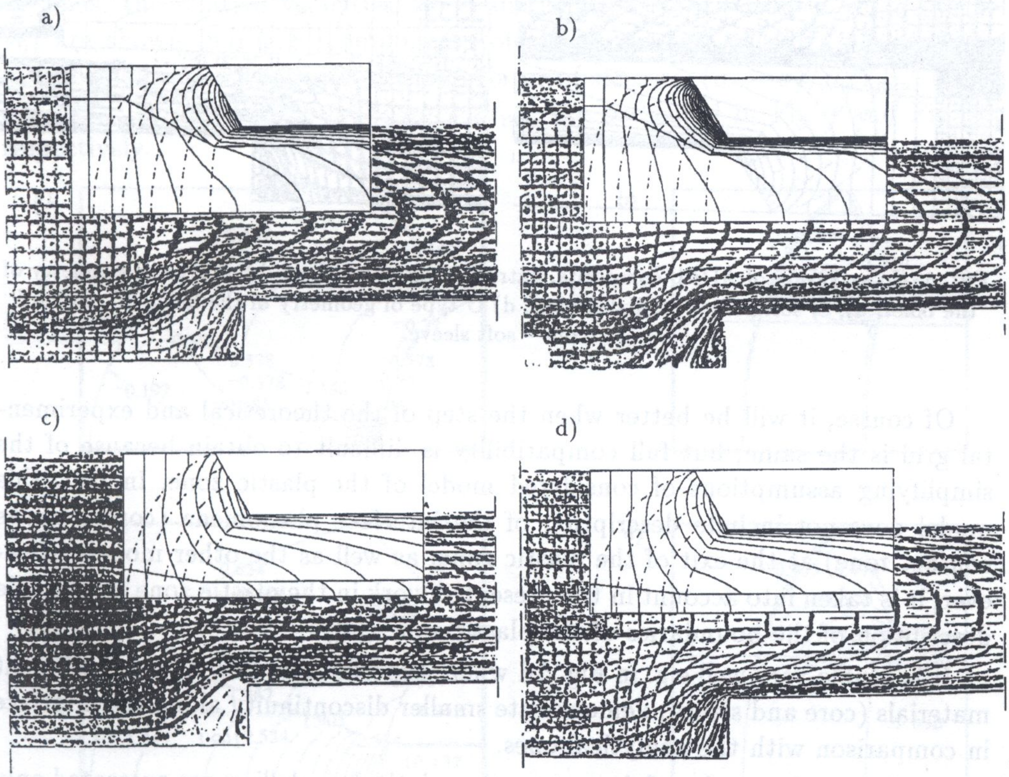


FIG. 5. Co-extrusion of lead/hard lead alloy composite, extrusion ratio $\lambda = 3$: a), b) *A* type of geometry of the billet; a), c) soft core – hard sleeve; c), d) *C* type of geometry of the billet; b), d) hard core – soft sleeve.

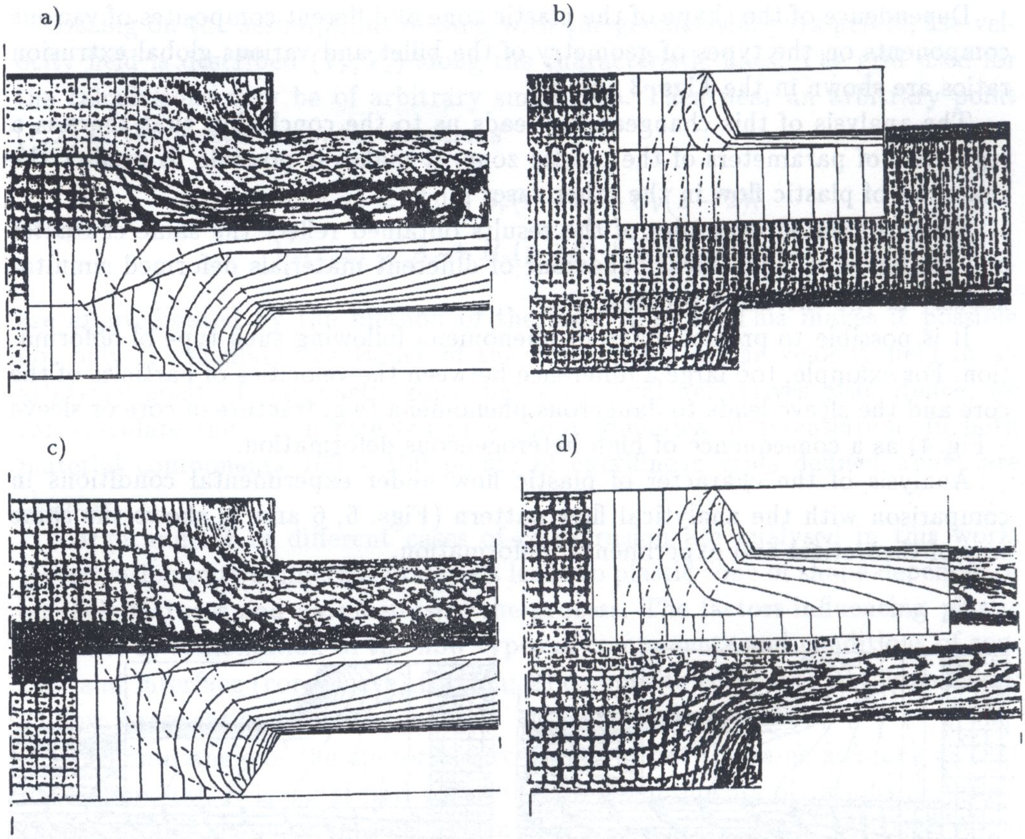


FIG. 6. Co-extrusion of Al/Pb composite, extrusion ratio $\lambda = 3$. a), b) B type of geometry of the billet; a), c) soft core – hard sleeve; c), d) C type of geometry of the billet; b), d) hard core – soft sleeve.

Of course, it will be better when the step of the theoretical and experimental grid is the same, but full compatibility is difficult to obtain because of the simplifying assumptions of considered model of the plastic zone. In fact, this model does not include description of the interface phenomena (core – sleeve contact zone) at the exit of the plastic zone, as well as the other models. However, it is taken into account in the presented work in the plastic zone by suitable description of the boundaries of the plastic zone ([4]).

It is visible very well e.g. in Fig. 5 d where experimental flow lines for different materials (core and sleeve) demonstrate smaller discontinuity along the interface in comparison with the analytical ones.

Further on the results of the proposed analytical modelling are presented only for two cases (hard core – soft sleeve and soft core – hard sleeve) of co-extrusion of Al/Pb composite of the type B geometry (see Fig. 4), as a representation

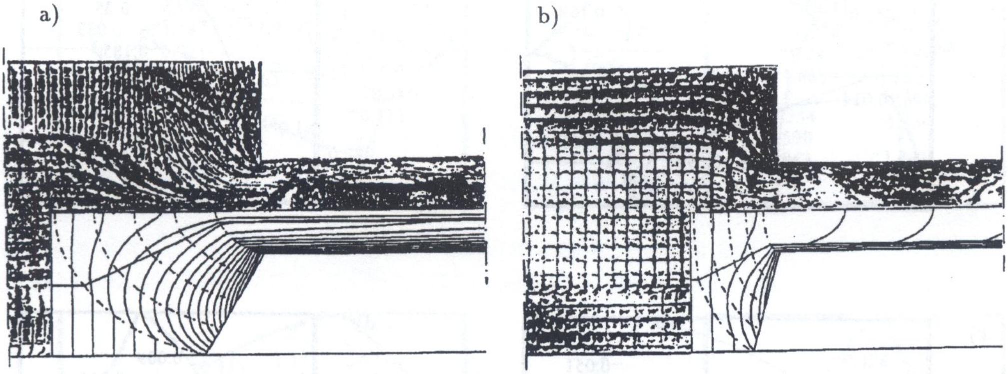


FIG. 7. Co-extrusion of Al/Pb composite, extrusion ratio $\lambda = 12$, B type of the geometry of billet: a) soft core – hard sleeve; b) hard core – soft sleeve.

of all kinds of results obtained in the proposed model of the plastic flow. But conclusions are drawn taking into account all cases which are not presented here.

Thus, the relative velocities V_r, V_z distributions (with respect to the value V_0) are shown in Fig. 8; components of the strain rate tensor $\dot{\varepsilon}_r, \dot{\varepsilon}_z, \dot{\varepsilon}_{rz}, \dot{\varepsilon}_\theta$; and normalized components of the deviator of stresses: $(\sigma_r - \sigma)/\sqrt{2}Y_{\text{min}}$, $(\sigma_z - \sigma)/\sqrt{2}Y_{\text{min}}$, $\sigma_{rz}/\sqrt{2}Y_{\text{min}}$, $(\sigma_\theta - \sigma)/\sqrt{2}Y_{\text{min}}$ are presented in Fig. 9 and Fig. 10, respectively.

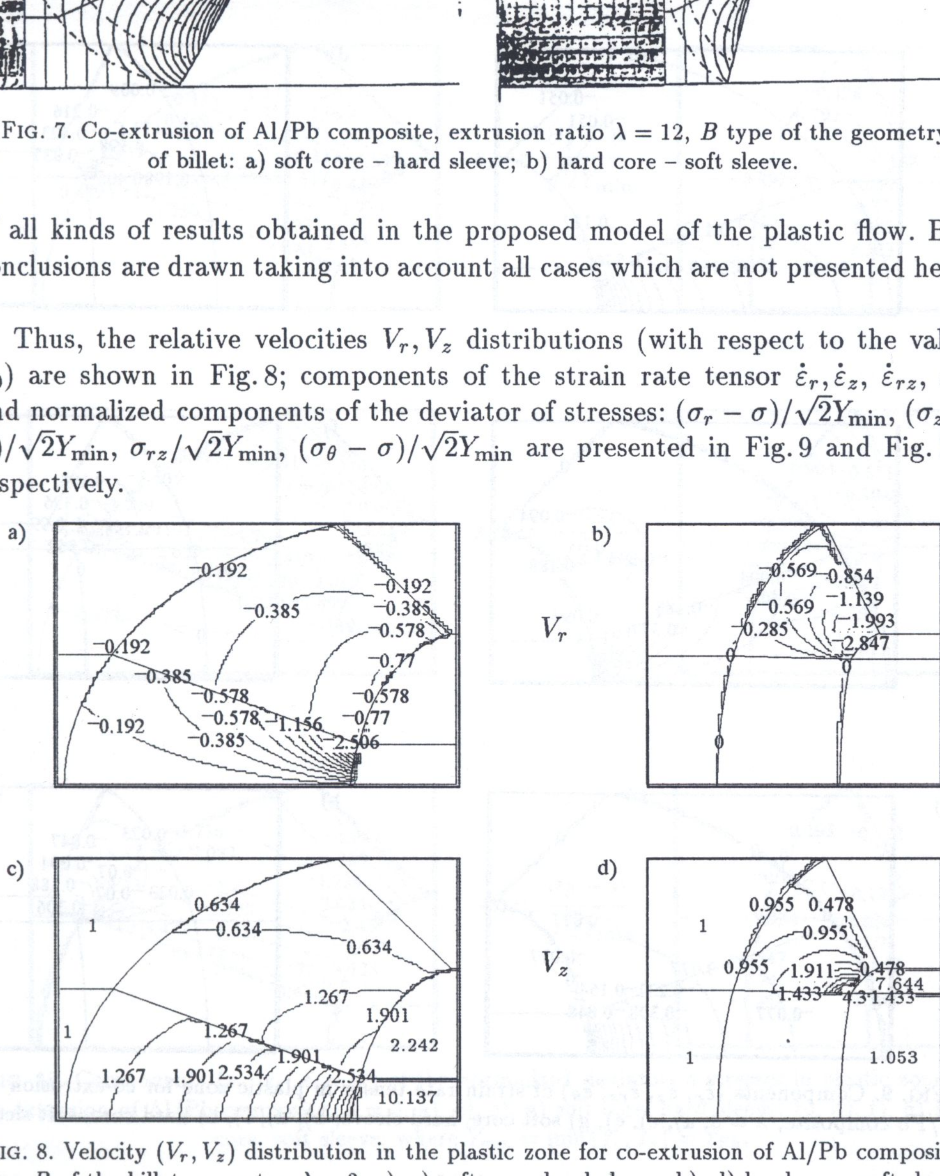


FIG. 8. Velocity (V_r, V_z) distribution in the plastic zone for co-extrusion of Al/Pb composite, type B of the billet geometry, $\lambda = 3$, a), c) soft core, hard sleeve; b), d) hard core, soft sleeve.

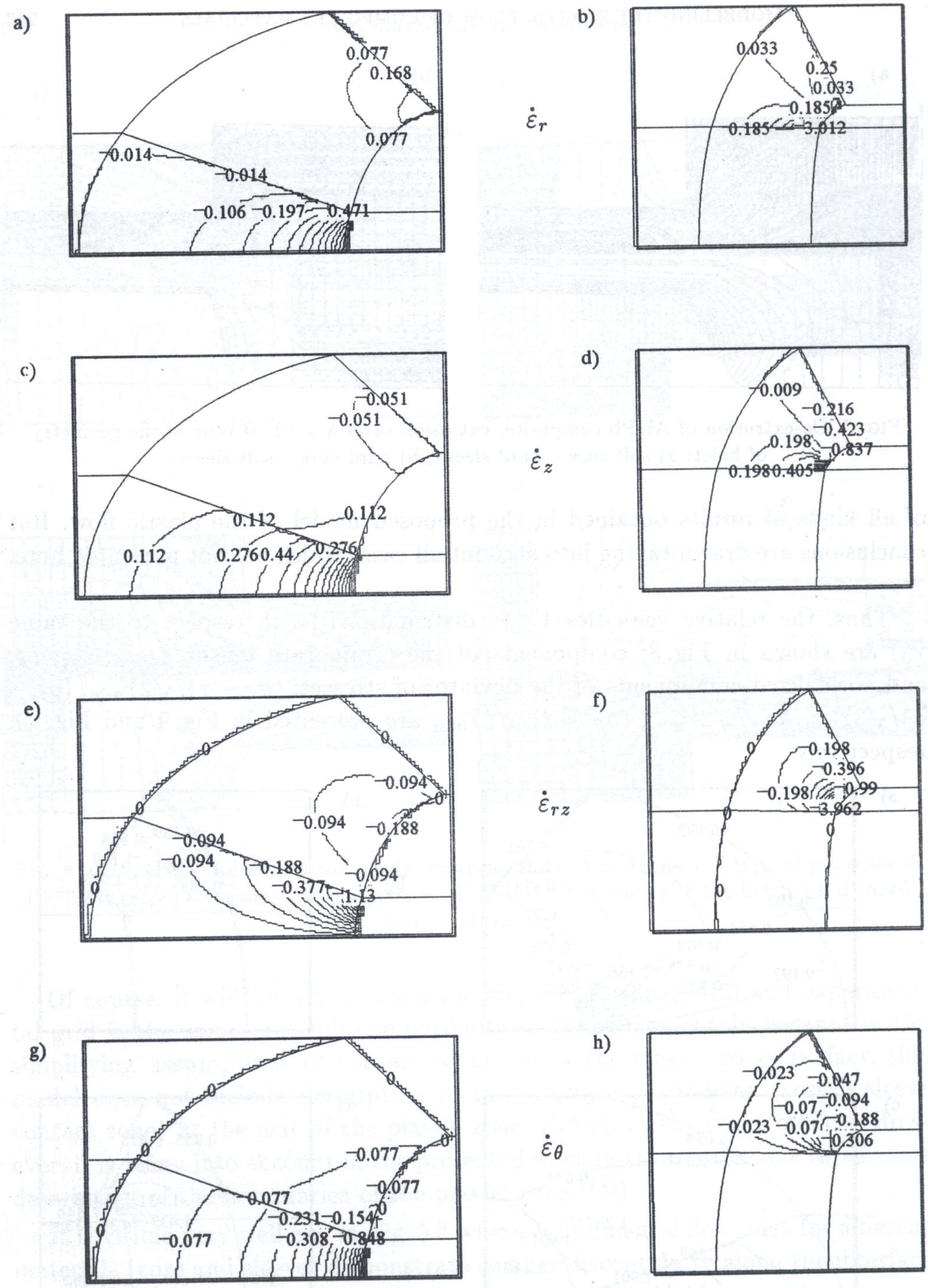
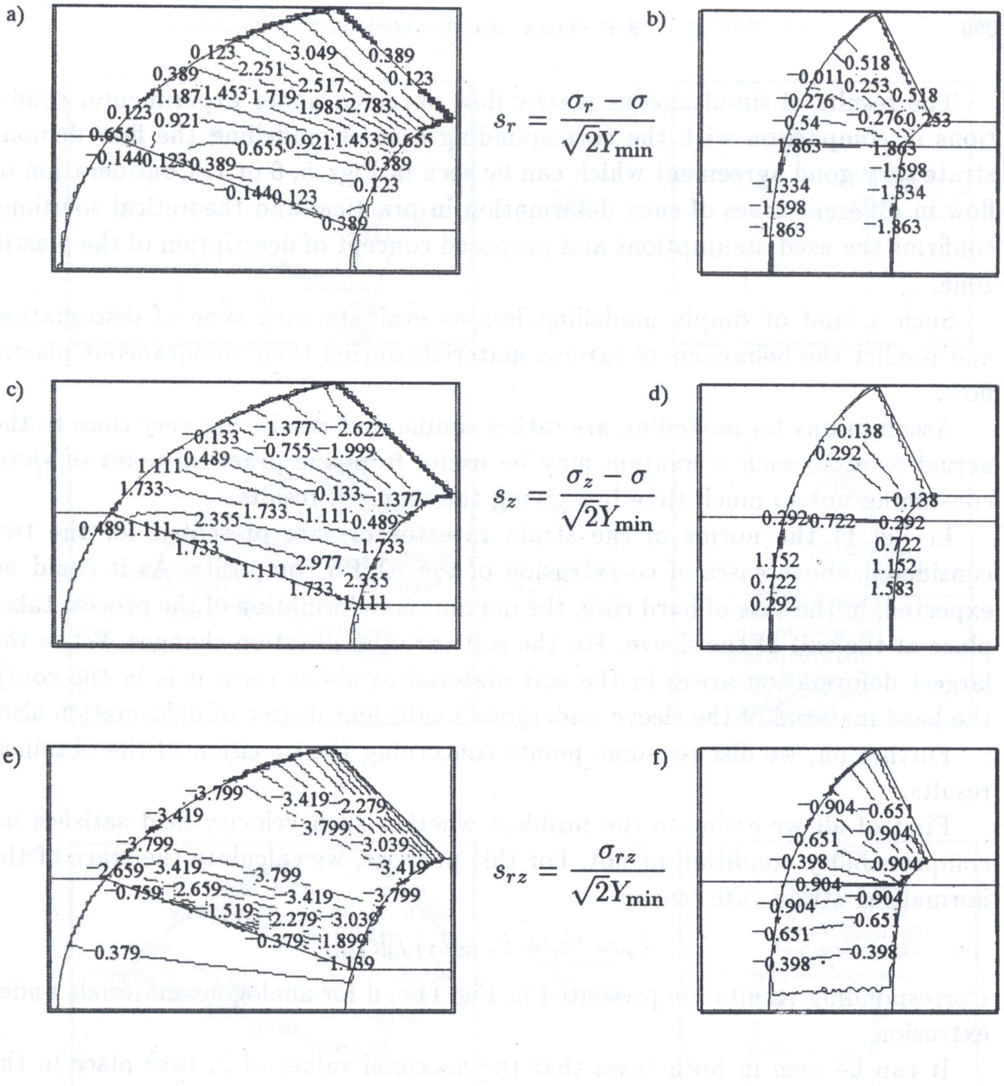


FIG. 9. Components ($\dot{\varepsilon}_r$, $\dot{\varepsilon}_z$, $\dot{\varepsilon}_{rz}$, $\dot{\varepsilon}_\theta$) of strain rate tensor in plastic zone for co-extrusion of Al/Pb composite, $\lambda = 3$, a), c), e), g) soft core, hard sleeve; b), d), f), h) hard core, soft sleeve.



$$s_r = \frac{\sigma_r - \sigma}{\sqrt{2}Y_{\min}}$$

$$s_z = \frac{\sigma_z - \sigma}{\sqrt{2}Y_{\min}}$$

$$s_{rz} = \frac{\sigma_{rz}}{\sqrt{2}Y_{\min}}$$

$$s_\theta = \frac{\sigma_\theta - \sigma}{\sqrt{2}Y_{\min}}$$

FIG. 10. Components (s_r, s_z, s_{rz}, s_θ) of the normalized deviator of stresses in plastic zone for co-extrusion of Al/Pb composite, $\lambda = 3$, a), c), e), g) soft core, hard sleeve; b), d), f), h) hard core, soft sleeve; where $Y_{\min} = \min\{Y_c, Y_s\} = Y_{Pb}$.

The results of simultaneous plastic flow obtained under experimental conditions in comparison with the corresponding case of modelling the flow demonstrate very good agreement which can be seen in Figs. 5, 6 or 7. Consideration of flow in different cases of such deformation in practical and theoretical solutions confirms the used assumptions and proposed concept of description of the plastic zone.

Such a kind of simple modelling lets us evaluate such type of deformation and predict the behaviour of various materials during their simultaneous plastic flow.

Assumptions for modelling are rather simple, but results are very close to the actual ones, so such a solution may be useful from the practical point of view, consuming not so much time but giving the required results.

In Fig. 11 the norms of the strain rate tensors are presented for the two considered above cases of co-extrusion of the Al/Pb composite. As it could be expected, in the case of hard core, the maximum deformation of the process takes place at the exit of the sleeve. For the soft core the situation changes. While the largest deformation arises in the soft material as above (now it is in the core), the hard material of the sleeve undergoes a sufficient degree of deformation also.

Further on, we discuss some points concerning the precision of the obtained results.

First of all we estimate the problem whether such velocity field satisfies incompressibility condition or not. For this purpose, we calculate the trace of the normalized strain rate tensor:

$$\dot{\varepsilon}_* = (\dot{\varepsilon}_r + \dot{\varepsilon}_z + \dot{\varepsilon}_\theta) / ||\dot{\varepsilon}_{ij}||.$$

Corresponding results are presented in Fig. 11 c, d for analogous materials under extrusion.

It can be seen in both cases that the maximal values of $\dot{\varepsilon}_*$ take place in the sleeve exit. This error is not connected with the accuracy of the calculations. There is no important difference between the results for large (100×100) or small (30×30) number of points (in the core and the sleeve, separately) in the approximation of the flow. From the author's point of view, this mistake deals with the Assumption **5** of the straight lines. Nevertheless, the values of the relative error (10% in the worse case) are still tolerable, because the approximation model is still sufficiently simple.

The next point which could be discussed is the possibility to apply the relation:

$$\sigma_r = \sigma_\theta , \tag{4.2}$$

to the whole plastic zone. Such an assumption is very popular. It was proposed in [13], and often used in the engineering approach (see [14]). An analogous

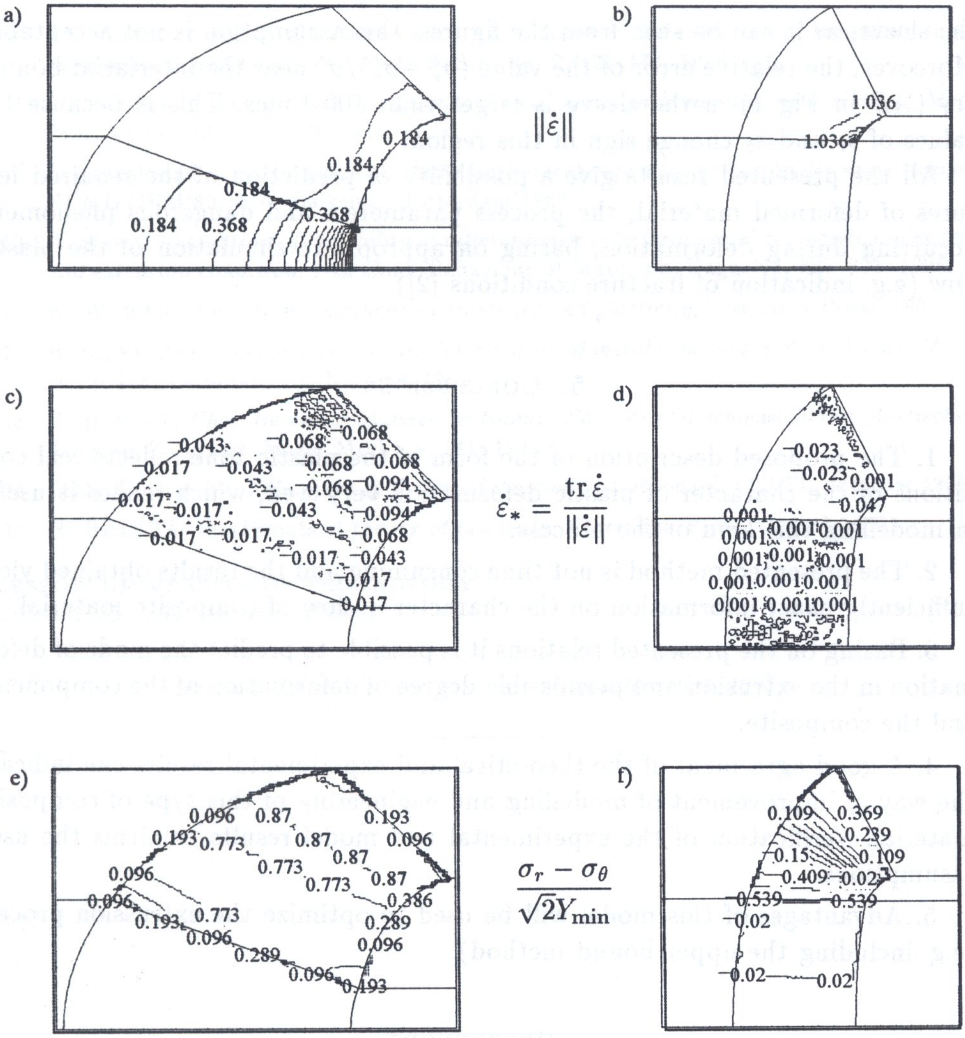


FIG. 11. Co-extrusion of Al/Pb composite, $\lambda = 3$. a), c), e) soft core, hard sleeve; b), d), f) hard core, soft sleeve; a), b) distribution of the norm of the strain rate tensor; c), d) estimations of the normalized trace ε_* of the tensor $\dot{\varepsilon}$; c), d) estimations of the simplified assumption: $\sigma_r = \sigma_\theta$.

assumption is presently used in [6, 7]. However, as it has been stressed in [14, 15], there are arguments against the Assumption (4.2). In Fig. 11 e, f the normalized values of

$$\sigma_* = (\sigma_r - \sigma_\theta)/\sqrt{2}Y_{\min}$$

are presented, where $Y_{\min}$ is the smaller value of Y_c or Y_s. As it should be expected, the assumption (4.2) is in a good agreement with the result obtained in the core (near the axis OZ this assumption is theoretically justified). But in the sleeve, as it can be seen from the figures, this assumption is not acceptable. Moreover, the relative error of the value $(\sigma_r' - \sigma_\theta')/\sigma_r'$ near the interfacial boundary (AB in Fig. 1) in the sleeve is larger than 100 times. This is because the values of s_r and s_θ change sign in this region.

All the presented results give a possibility of prediction of the required features of deformed material, the process parameters and dangerous phenomena occurring during deformation, basing on appropriate simulation of the plastic flow (e.g. indication of fracture conditions [2]).

5. Conclusions

1. The proposed description of the form of the plastic zone reflects real conditions of the character of plastic deformation very well, which makes it useful in modelling this kind of the process.

2. The presented method is not time consuming and the results obtained yield sufficiently exact information on the character of flow of composite material.

3. Basing on the presented relations it is possible to predict the mode of deformation in the extrusion and permissible degree of deformation of the components and the composite.

4. A good agreement of the theoretical and experimental results can indicate the way of improvement of modelling and engineering of this type of composite material. Verification of the experimental and model results confirms the used assumptions.

5. Advantages of this model will be used to optimize the extrussion process (e.g. including the upper bound method).

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