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POLAR FLOW PAST A SPHEROID

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The problem of symmetrical Stokes flow of an incompressible micropolar fluid past a spheroid is considered in the paper. The shape of the spheroid differs slightly from that of a sphere. An explicit expression is obtained for the stream function. The boundary conditions at the spheroid surface allow for a slip of tangential fluid flow velocity.

1. Introduction

The problem of slow, steady symmetrical flow due to the translation of an approximately spherical solid particle in an unbounded fluid medium was first investigated in 1891 by SAMPSON [1]. Since then BRENNER [2] and ACRIVOS and TAYLOR [3] have independently examined the asymmetric case. In all instances the authors assumed the no-slip condition and considered a Newtonian fluid medium. There are, however, situations where there is some slip at the surface and the fluid medium is non-Newtonian. It is one of these situations that we now address ourselves.

In this note the problem of symmetrical micropolar fluid flow past a spheroid whose shape differs slightly from that of a sphere, is examined under the assumption of slip at the surface. An explicit expression is obtained for the stream function associated with the flow field to the first order in the small parameter characterizing the deformation. As an application, we consider micropolar flow past an oblate spheroidal particle and derive the drag experienced by it. Special known cases including flow past a perfect sphere with no-slip on its surface [4] are deduced.

2. Statement and solution of the problem

We consider the case of symmetrical Stokes flow of an incompressible micropolar fluid past a spheroid whose shape varies slightly from that of a sphere, and which is held fixed in an otherwise uniform stream of speed U in the absence of body forces and couples. We refer the motion to a spherical coordinate system

(r,θ,φ). The stream function characterizing the micropolar flow field is given by [5]:

$$\psi(r,\theta)=\sum_{n=2}^{\infty}\Big[A_n r^n + B_n r^{-n+1} + C_n r^{n+2} + D_n r^{-n+3} + E_n r^{1/2} K_{n-1/2}(\lambda r)\Big] I_n(\zeta), \tag{2.1}$$

where $\zeta=\cos\theta$, $\lambda^2=\dfrac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}$ with (μ,κ,γ) being micropolar material constants and $I_n(\zeta)$ is the Gegenbauer function connected with the Legendre function $P_n(\zeta)$ by the relation

$$I_n(\zeta)=\frac{P_{n-2}(\zeta)-P_n(\zeta)}{2n-1}, \qquad n\ge 2.$$

These functions have the following special property [6]:

$$I_m I_2 = -\frac{(m-2)(m-3)}{2(2m-1)(2m-3)} I_{m-2} + \frac{m(m-1)}{(2m+1)(2m-3)} I_m - \frac{(m+1)(m+2)}{2(2m-1)(2m+1)} I_{m+2}, \qquad m\ge 2. \tag{2.2}$$

As usual, the stream function ψ is related to the velocity field $(u_r, u_\theta, 0)$ by

$$u_r=-\frac{I}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}, \qquad u_\theta=\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}. \tag{2.3}$$

We take the surface S approximating that of the sphere to be of the form $r=a[1+f(\theta)]$. The orthogonality of the Gegenbauer functions enables us, under general circumstances, to assume the expansion $f(\theta)=\sum_{k=1}^{\infty}\alpha_k I_k(\zeta)$. We can therefore take S to be

$$r=a[1+\alpha_m I_m(\zeta)]$$

and neglect the terms of $O(\alpha_m^2)$.

Our main problem here is to determine the flow field in which case we need to obtain the velocity field and the independent microrotation field $\boldsymbol{\nu}$ which micropolar fluid theory admits [7]. It can be shown [5] that for our particular problem $\boldsymbol{\nu}=(0,0,\nu_\varphi)$, where

$$\frac{a\nu_\varphi}{U}=\frac{1}{\sigma\sqrt{1-\zeta^2}}\Bigg\{\Big[e_2\lambda^2 a^2\frac{(\mu+\kappa)}{\kappa}\sigma^{1/2}\kappa_{3/2}(\lambda a\sigma)-\frac{d_2}{\sigma}\Big] I_2(\zeta) + \sum_{n=3}^{\infty}\Big[E_n\lambda^2 a^2\frac{\mu+\kappa}{\kappa}\sigma^{1/2}K_{n-1/2}(\lambda a\sigma)-(2n-3)D_n\Big(\frac{1}{\sigma}\Big)^{n-1}\Big] I_n(\zeta)\Bigg\}. \tag{2.4}$$

We write the stream function in the dimensionless form

$$\frac{\psi}{Ua^2} = \left[a_2\sigma^2 + \frac{b_2}{\sigma} + c_2\sigma^4 + d_2\sigma + e_2\sigma^{1/2}K_{3/2}(\lambda a\sigma)\right] I_2(\zeta) + \sum_{n=3}^{\infty}\Big[A_n\sigma^n + B_n\sigma^{-n+1} + C_n\sigma^{n+2} + D_n\sigma^{-n+3} + E_n\sigma^{1/2}K_{n-1/2}(\lambda a\sigma)\Big] I_n(\zeta), \tag{2.5}$$

where $\sigma = r/a$. Using the fact that the component of the velocity at the origin must be finite and that

$$\psi \to \frac{1}{2}Ur^2\sin^2\theta \qquad \text{as} \quad r \to \infty, \tag{2.6}$$

Eq. (2.5) reduces to

$$\frac{\psi}{Ua^2} = \left[\sigma^2 + \frac{b_2}{\sigma} + d_2\sigma + e_2\sigma^{1/2}K_{3/2}(\lambda a\sigma)\right] I_2(\zeta) + \sum_{n=3}^{\infty}\left[B_n\sigma^{-n+1} + D_n\sigma^{-n+3} + E_n\sigma^{1/2}K_{n-1/2}(\lambda a\sigma)\right] I_n(\zeta). \tag{2.7}$$

In the case of flow past a perfect sphere the only coefficients which contribute [4] are b_2, d_2 and e_2. Consequently, all other coefficients must be of $O(\alpha_m)$. Hence except where b_2, d_2 and e_2 are encountered, we may take the surface to be $\sigma = 1$.

In order to determine the flow field, we must now proceed to determine the unknown coefficients appearing in (2.4) and (2.7) and this we do with the aid of the boundary conditions.

The kinematic condition of mutual impenetrability at the surface S requires that we take

$$\psi = 0 \qquad \text{on} \quad S. \tag{2.8}$$

We assume the usual no-spin [7] condition. That is,

$$\nu_\varphi = 0 \qquad \text{on} \quad S. \tag{2.9}$$

As regards the slip condition, we use the most plausible hypothesis [8] that the tangential velocity of the fluid relative to the solid at a point on its surface is proportional to the tangential stress, $t_{r\theta}$, prevailing at that point. In our case this hypothesis takes the form

$$t_{r\theta} = \beta\mu_\theta \qquad \text{on} \quad S, \tag{2.10}$$

where the constant β is the coefficient of sliding friction. It can be shown [5] that in terms of the stream function (2.10) can be written as

$$-2(\mu+k)\frac{\partial\psi}{\partial r}+(\mu+k)r\frac{\partial^2\psi}{\partial r^2}=\beta r\frac{\partial\psi}{\partial r} \qquad \text{on } S. \tag{2.11}$$

These boundary conditions (2.8), (2.9) and (2.11) lead respectively to the following equations:

$$\begin{aligned} 0 = {} & [1+b_2+d_2+(1+\lambda a)f_2]\, I_2(\zeta) \\ & + \left[2-b_2+d_2-(1+\lambda a+\lambda^2a^2)f_2\right] \alpha_m I_2(\zeta)I_m(\zeta) \\ & + \sum_{n=3}^{\infty}\left[B_n+D_n+K_{n-1/2}(\lambda a)E_n\right] I_n(\zeta), \end{aligned} \tag{2.12}$$

$$\begin{aligned} 0 = {} & \left[-d_2+\lambda^2a^2\frac{(\mu+k)}{k}(1+\lambda a)f_2\right] I_2(\zeta) \\ & + \left[d_2-\lambda^2a^2\frac{(\mu+k)}{k}(1+\lambda a+\lambda^2a^2)f_2\right] \alpha_m I_2(\zeta)I_m(\zeta) \\ & + \sum_{n=3}^{\infty}\left[\lambda^2a^2\frac{(\mu+k)}{K}K_{n-1/2}(\lambda a)E_n-(2n-3)D_n\right] I_n(\zeta), \end{aligned} \tag{2.13}$$

$$\begin{aligned} 0 = {} & \Big[-2(1+\theta_1)+b_2(1+4\theta_1+3\theta_2)-d_2(1+2\theta_1+\theta_2) \\ & + f_2\Big\{(1+\lambda a+\lambda^2a^2)(1+2\theta_1+\theta_2) \\ & + (\theta_1+\theta_2)(2+2\lambda a+\lambda^2a^2+\lambda^3a^3)\Big\}\Big] I_2(\zeta) \\ & + \Big[-2-b_2(2+12\theta_1+9\theta_2)+d_2(2\theta_1+\theta_2) \\ & - f_2\Big\{(2+3\theta_2)(1+\lambda a)+\lambda^2a^2(1-\theta_1+\theta_2)+\lambda^3a^3(1+3\theta_1+2\theta_2) \\ & + \lambda^4a^4(\theta_1+\theta_2)\Big\}\Big] \alpha_m I_2(\zeta)I_m(\zeta) \\ & + \Big[(n-1)\{1+(2+n)\theta_1+(1+n)\theta_2\}B_n \\ & + (n-3)\{1+n\theta_1+(n-1)\theta_2\}D_n \\ & + \Big\{(n-1)k_{n-1/2}(\lambda a)(1+(2+n)\theta_1+(1+n)\theta_2) \\ & + \lambda a k_{n-3/2}(\lambda a)(1+(2n-1)\theta_1+(2n-2)\theta_2) \\ & + \lambda^2a^2(\theta_1+\theta_2)k_{n-5/2}\Big\} E_n\Big] I_n(\zeta), \end{aligned} \tag{2.14}$$

where $\theta_1=\mu/\beta a$ and $\theta_2=k/\beta a$ and $f_2=e_2\sqrt{\dfrac{\pi}{2\lambda^3a^3}}\,e^{-\lambda a}$.

The leading terms in the above equations (2.12)–(2.14) must vanish. Hence,

$$0 = 1 + b_2 + d_2 + (1+\lambda a) f_2 ,$$

$$0 = -d_2 + \lambda^2 a^2 \left(\frac{u+k}{k}\right)(1+\lambda a) f_2 ,$$

$$\begin{aligned}0 = &-2(1+\theta_1) + (1+4\theta_1+3\theta_2) b_2 - (1+2\theta_1+\theta_2) d_2 \\ &+\Big\{(1+\lambda a+\lambda^2 a^2)(1+2\theta_1+\theta_2) \\ &+(\theta_1+\theta_2)(2+2\lambda a+\lambda^2 a^2+\lambda^3 a^3)\Big\} f_2 .\end{aligned} \tag{2.15}$$

Solving the above systems yields

$$b_2 = \frac{1}{\lambda^2 a^2 \Delta}\Big[(3k+6k\theta_1+3k\theta_2) + \lambda a(3k+6\theta_1 k+3\theta_2 k) \\ +\lambda^2 a^2 (\mu+2k+3\theta_1 k-\theta_2 u+\theta_2 k) + \lambda^3 a^3 (u+k+\theta_1 k-\theta_2 \mu)\Big],$$

$$d_2 = -\frac{3}{\Delta}(u+k+\lambda a u+\lambda a k)(1+2\theta_1+\theta_2), \tag{2.16}$$

$$f_2 = -\frac{3k}{\lambda^2 a^2 \Delta}(1+2\theta_1+\theta_2) \qquad \text{or}$$

$$e_2 = -\frac{3ke^{\lambda a}}{\Delta}\sqrt{\frac{2}{\pi\lambda a}}(1+2\theta_1+\theta_2),$$

where

$$\Delta = (k+3k\theta_1+2k\theta_2+2\mu+6\theta_1\mu+4\theta_2\mu) \\ +\lambda a\,(5\theta_1 k+3\theta k_2+2\mu+2k+6\theta_1\mu+4\theta_2\mu) .$$

Substituting these values into the system (2.12)–(2.14) we obtain

$$\begin{aligned}
0 &= \beta_1 \alpha_m I_m(\zeta) I_{(2)}(\zeta) + \sum_{n=3}^{\infty}\Big[B_n + D_n + K_{n-1/2}(\lambda a) E_n\Big] I_n(\zeta), \\
0 &= \beta_2 \alpha_m I_m(\zeta) I_{(2)}(\zeta) \\
&\quad + \sum_{n=3}^{\infty}\Big[(3-2n) D_n + \lambda^2 a^2 \frac{(u+k)}{k} K_{n-1/2}(\lambda a)\Big] I_n(\zeta), \\
0 &= \beta_3 \alpha_m I_m(\zeta) I_{(2)}(\zeta) + \sum_{n=3}^{\infty}\left[\gamma_{1n} B_n + \gamma_{2n} D_n + \gamma_{3n} E_n\right] I_n(\zeta),
\end{aligned} \tag{2.17}$$

where

$$\beta_1 = 2 - b_2 + d_2 - \left(1 + \lambda a + \lambda^2 a^2\right) f_2 ,$$

$$\beta_2 = -\lambda^4 a^4 \frac{u+k}{k} ,$$

$$\beta_3 = -2 - (2 + 12\theta_1 + 9\theta_2)\, b_2 + (2\theta_1 + \theta_2)\, d_2 - \Big\{(2 + 3\theta_2)(1 + \lambda a) + \lambda^2 a^2 (1 - \theta_1 + \theta_2) + \lambda^3 a^3 (1 + 3\theta_1 + 2\theta_2) + \lambda^4 a^4 (\theta_1 + \theta_2)\Big\} f_2 ,$$

(2.18)
$$\gamma_{1n} = (n-1)\{1 + (2+n)\theta_1 + (1+n)\theta_2\} ,$$

$$\gamma_{2n} = (n-3)\{1 + n\theta_1 + (n-1)\theta_2\} ,$$

$$\gamma_{3n} = (n-1)\{1 + (2+n)\theta_1 + (1+n)\theta_2\}\, k_{n-1/2}(\lambda a) + \lambda a\{1 + (2n-1)\theta_1 + (2n-2)\theta_2\}\, k_{n-3/2}(\lambda a) + \lambda^2 a^2 (\theta_1 + \theta_2) k_{n-5/2}(\lambda a),$$

with b_2, d_2, f_2 being given in (2.16).

Solving the system (2.17) with the aid of (2.2), we see that all the coefficients vanish except those for which n takes the values $m-2$, m and $m+2$, and for these coefficients we get the following expressions:

$$B_{m-2} = \Big[\frac{-\overline{a}_m}{\Delta_1} k\gamma_3 (\beta_2 - 7\beta_1 + 2\beta_1 m) + k_{m-5/2}(\lambda a)\Big\{-k\gamma_2\beta_2 + \lambda^2 a^2 \mu\gamma_2\beta_1 + \lambda^2 a^2 k\gamma_2\beta_1 + 7\beta_3 k - 2\beta_3 k m - \lambda^2 a^2 \mu\beta_3 - \lambda^2 a^2 k\beta_3\Big\}\Big],$$

$$D_{m-2} = \frac{\overline{a}_m}{\Delta_1}\Big[k\gamma_3\beta_2 + K_{m-5/2}(\lambda a)\Big\{\lambda^2 a^2 \mu\beta_1\lambda_1 + \lambda^2 a^2 k\beta_1\gamma_1 - \lambda^2 a^2 \mu\beta_3 - \lambda^2 a^2 k\beta_3 - k\beta_2\gamma_1\Big\}\Big],$$

(2.19)
$$E_{m-2} = \frac{k\overline{a}_m}{\Delta_1}\left[7\beta_3 - 2m\beta_3 - \beta_2\gamma_2 + \beta_2\gamma_1 - 7\beta_1\gamma_1 + 2\beta_1\gamma_1 m\right],$$

$$B_m = \frac{-\overline{b}m}{\Delta_2}\Big[k\delta_3(\beta_2 - 3\beta_1 + 2\beta_1 m) + k_{m-1/2}\Big\{-k\delta_2\beta_2 + \lambda^2 a^2 \mu\delta_2\beta_1 + \lambda^2 a^2 k\delta_2\beta_1 + 3\beta_3 k - 2\beta_3 k m - \lambda^2 a^2 u\beta_3 - \lambda^2 a^2 k\beta_3\Big\}\Big],$$

$$D_m = \frac{\overline{b}m}{\Delta_2}\Big[k\delta_3\beta_2 + k_{m-1/2}(\lambda a)\Big\{\lambda^2 a^2 u\beta_1\delta_1 + \lambda^2 a^2 k\beta_1\delta_1 - \lambda^2 a^2 u\beta_3 - \lambda^2 a^2 k\beta_3 - k\beta_2\delta_1\Big\}\Big],$$

(2.19) [cont.]
$$E_m = \frac{k\bar{b}_m}{\Delta_2}\left[3\beta_3 - 2\beta_3 m - \beta_2\delta_2 + \beta_2\delta_1 - 3\beta_1\delta_1 + 2\beta_1\delta_1 m\right],$$

$$B_{m+2} = \frac{-\bar{c}_m}{\Delta_3}\Big[k\sigma_3(\beta_2 + \beta_1 + 2\beta_1 m) + K_{m+3/2}(\lambda a)\Big\{-k\sigma_2\beta_2 + \lambda^2 a^2\mu\sigma_2\beta_1 + \lambda^2 a^2 k\sigma_2\beta_1 - \beta_3 k - 2\beta_3 k_m - \lambda^2 a^2\mu\beta_3 - \lambda^2 a^2 k\beta_3\Big\}\Big],$$

$$D_{m+2} = \frac{\bar{c}_m}{\Delta_3}\Big[k\sigma_3\beta_2 + k_{m+3/2}(\lambda a)\Big\{\lambda^2 a^2\mu\beta_1\sigma_1 + \lambda^2 a^2 k\beta_1\sigma_1 - \lambda^2 a^2\mu\beta_3 - \lambda^2 a^2 k\beta_3 - k\beta_2\sigma_1\Big\}\Big],$$

$$E_{m+2} = \frac{k\bar{c}_m}{\Delta_3}\left[-\beta_3 - 2\beta_3 m - \beta_2\sigma_2 + \sigma_1\beta_2 + \sigma_1\beta_1 + 2\beta_1\sigma_1 m\right],$$

where

$$\bar{a}_m = \frac{\alpha_m(m-2)(m-3)}{2(2m-1)(2m-3)},$$

$$\bar{b}_m = \frac{-\alpha_m m(m-1)}{(2m+1)(2m-3)},$$

$$\bar{c}_m = \frac{\alpha_m(m+1)(m+2)}{2(2m-1)(2m+1)},$$

$$\gamma_i = \gamma_{i(m-2)}, \qquad \delta_i = \delta_{im},$$

$$\sigma_i = \sigma_{i(m+2)} \qquad (i = 1, 2, 3),$$

$$\Delta_1 = 7k\gamma_3 - 2km\gamma_3 + k_{m-5/2}(\lambda a)\Big\{-7k\gamma_1 + 2km\gamma_1 - \lambda^2 a^2\mu\gamma_2 - \lambda^2 a^2 k\gamma_2 + \lambda^2 a^2\mu\gamma_1 + \lambda^2 a^2 k\gamma_1\Big\},$$

$$\Delta_2 = 3k\sigma_3 - 2km\delta_3 + k_{m-1/2}(\lambda a)\Big\{-3k\delta_1 + 2km\delta_1 - \lambda^2 a^2\mu\delta_2 - \lambda^2 a^2 k\delta_2 + \lambda^2 a^2\mu\delta_1 + \lambda^2 a^2 k\delta_1\Big\},$$

$$\Delta_3 = -k\sigma_3 - 2km\sigma_3 + k_{m+3/2}(\lambda a)\Big\{k\sigma_1 + 2km\sigma_1 - \lambda^2 a^2\mu\sigma_2 - \lambda^2 a^2 k\sigma_2 + \lambda^2 a^2\mu\sigma_1 + \lambda^2 a^2 k\sigma_1\Big\}.$$

To summarize, we have determined the stream function in the case of slip polar flow past a slightly deformed sphere. It is given by

$$(2.20)\quad \frac{\psi}{Ua^2} = \left[\sigma^2 + \frac{b_2}{\sigma^2} + d_2\sigma + (1+\lambda a)\sigma^{1/2} f_2\right] I_2(\zeta) + \left[B_{m-2}\sigma^{-m+3} + D_{m-2}\sigma^{-m+5} + E_m\sigma^{1/2}k_{m-5/2}(\lambda a\sigma)\right] I_{m-2}(\zeta) + \left[B_m\sigma^{-m+1} + D_m\sigma^{-m+3} + E_m\sigma^{1/2}k_{m-1/2}(\lambda a\sigma)\right] I_m(\zeta) + \left[B_{m+2}\sigma^{-m-1} + D_{m+2}\sigma^{-m+1} + E_{m+2}\sigma^{1/2}k_{m+3/2}(\lambda a\sigma)\right] I_{m+2}(\zeta),$$

where the constants are given above.

3. Application to an oblate spheroid

As an application of the foregoing analysis we now consider the particular case of slip polar flow past an oblate spheroid whose equation we take as

$$(3.1)\quad \frac{x^2+y^2}{c^2} + \frac{z^2}{c^2(1-\varepsilon^2)} = 1.$$

As before, we neglect terms of $O(\varepsilon^2)$. We rewrite (3.1) in the polar form

$$r = a\left[1 + 2\varepsilon I_2(\zeta)\right] \qquad \text{or} \qquad \sigma = 1 + 2\varepsilon I_2(\zeta),$$

where $a = c(1-\varepsilon)$. It follows that we must take $\alpha_m = 2\varepsilon$ and $m = 2$. Substitution into (2.19) and (2.20) gives the associated stream function as

$$(3.2)\quad \psi = Uc^2\left[\left(\frac{r}{c}\right)^2 + \left(\frac{c}{2}\right)\{b_2(1-3\varepsilon) + B_2\} + \left(\frac{r}{c}\right)\{d_2(1-\varepsilon) + D_2\} + \left(\frac{r}{c}\right)^{1/2} k_{3/2}(\lambda r)\left\{e_2\left(1 - \frac{3}{2}\varepsilon\right) + E_2\right\}\right] I_2(\zeta) + Uc^2\left[B_4\left(\frac{c}{r}\right)^3 + D_4\left(\frac{c}{r}\right) + E_4\left(\frac{r}{c}\right)^{1/2} k_{7/2}(\lambda r)\right] I_4(\zeta),$$

where explicit expressions for the constants can be obtained from expressions given in the last section.

We now focus on an important physical feature of the flow – the force experienced by the spheroid. Evaluation of this drag D, is most readily obtained by the application of the elegant formula [4]

$$(3.3)\quad D = 4\pi(2\mu + k)\lim_{r\to\infty} \frac{(\psi - \psi_\infty)}{r\sin^2\theta},$$

where ψ_∞ represents the stream function corresponding to the fluid motion at infinity.

Hence, $\psi_\infty = Ur^2 I_2(\zeta)$. Utilizing this and (3.2) in (3.3) gives

$$D = 2\pi(2\mu + k)Uc\{d_2(1-\varepsilon) + D_2\}, \tag{3.4}$$

where

$$d_2 = -\frac{A}{E} - \varepsilon\frac{(BE - AF)}{E^2}, \tag{3.5}$$

$$D_2 = -\frac{4\varepsilon}{5}\frac{\left[B_2\delta_3 k + k_{3/2}(\lambda c)\{\lambda^2c^2\delta_1\beta_1(u+k) - \lambda^2c^2\beta_3(u+k) - \delta_1\beta_2 k\}\right]}{-\delta_3 k + k_{3/2}(\lambda c)\{\lambda^2c^2\delta_1(u+k) - \lambda^2c^2\delta_2(u+k) + \delta_1 k\}}, \tag{3.6}$$

$$\begin{aligned}
A &= 3(u+k)(1+\lambda c)(1+2\phi_1+\phi_2),\\
B &= -3(u+k)(\lambda c - 2\phi_1 - \phi_2),\\
E &= (k + 2u + 3k\phi_1 + 2k\phi_2 + 6\phi_1 u + 4\phi_2 u)\\
&\qquad + \lambda c(5\phi_1 k + 3\phi_2 k + 2u + 2k + 6\phi_1 u + 4\phi_2 u),\\
F &= 3k\phi_1 + 2k\phi_2 + 6\phi_1\mu + 4\phi_2 u - 2\lambda c(u+k),\\
\phi_1 &= \frac{u}{\beta c}, \qquad \phi_2 = \frac{k}{\beta c}.
\end{aligned}$$

The following cases can now be deduced:

a. *No-slip flow past an oblate spheroid*

Here $\beta = \infty \Rightarrow \phi_1 = \phi_2 = 0$.
Substitution into (3.4)–(3.6) gives

$$D = 2\pi(u+k)Uc\left\{\frac{-3(u+k)(1+\lambda c)}{k + 2u + 2\lambda c(u+k)^2} + \varepsilon\left[3(u+k)\frac{(2\lambda^2c^2(u+k) - k - 2u)}{(k+2u+2\lambda c(u+k))^2} + \frac{4}{5}\tau\right]\right\}, \tag{3.7}$$

where

$$\tau = \left[\lambda^5c^5k_{1/2}(\lambda c) + k_{2/3}(\lambda c)\frac{(u+k)(k + k\lambda c + 2k\lambda^2c^2 - 2\mu\lambda^2c^2 - 2k\lambda^3c^3 - 2\mu\lambda^3c^3)}{k + 2\mu + 2\lambda c(u+k)}\right] \Big/ \left[\lambda c k_{1/2}(\lambda c) + 2\left\{k + \lambda^2c^2(u+k)\right\}k_{3/2}(\lambda c)\right].$$

This is a new result. In the case of a perfect sphere we recover the previously obtained result [4],

$$D = \frac{-6\pi Uc(2\mu + k)(u+k)(1+\lambda c)}{k + 2u + 2\lambda c(u+k)}. \tag{3.8}$$

b. *Slip flow past a sphere*

Here $\varepsilon = 0 \Rightarrow D_2 = 0$.
Substitution in the above gives the drag as

$$D = \frac{-6\pi U c(2u+k)(u+k)(\beta c+2u+k)(1+\lambda c)}{X+\lambda c Y}, \tag{3.9}$$

where

$$X = \beta c + 3ku + 2k^2 + 2u\beta c + 6u^2 + 4uk,$$
$$Y = 5uk + 3k^2 + 2\mu\beta c + 2k\beta c + 6u^2 + 4uk.$$

Again this is a new result. In the case of a classical Newtonian fluid ($k = 0$) this reduces to the well-known result [6],

$$D = -6\pi\mu U c\frac{\beta c + 2u}{\beta c + 3u}.$$

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