ENGINEERING TRANSACTIONS • *Engng. Trans.* • **60**, 1, 41–54, 2012
Polish Academy of Sciences • Institute of Fundamental Technological Research (IPPT PAN)
National Engineering School of Metz (ENIM)
SIXTY YEARS OF THE ENGINEERING TRANSACTIONS

Theoretical Basis of Determining the Translation and Rotation of Steering Wheel Stub Axle

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In the paper a theoretical basis of determination of the steering wheel stub axle position and orientation in the suspension movements space, realized by the suspension mechanism are presented. The designed instrument allows measurement of quantities that are needed to compute the translation and orientation of the stub axleand to draw the kinematic characteristics of the suspension.

Key words: passenger car, multi-link suspension, steering wheel translation and orientation.

1. Introduction

Analysis of the car movement parameters on the curved track is one of the fundamental issues of stability and steerability. A change of these parameters is always caused by a change of the external forces acting on the vehicle.

The car comparative studies carried out so far show that even at the same kinematic extortion, realized by the change of steering angle, the change of the forces generated at the wheel-road contact patch is dependent on many factors associated with tire, suspension, and steering system construction.

Steering wheels are carried out against the car body through the spatial mechanisms with flexible constraints. Flexibility is the reason that during a car ride along the same path, with different speeds, the real kinematic steering ratio is changing; a significant difference between the real and theoretical steering angles appears. Measurement of real steering and camber angles in experimental car studies has a significant value. Results of such measurements are used to work out the relationships between car movement parameters as well as for stability and steerability evaluation [4, 9].

In case of independent suspensions, wheel vertical movements caused by unevenness of the road surface cause a track change. It leads to wheels drifting and adversely affects straight driving [6, 11]. Measurement of position and orientation of the wheel relative to the car body is very difficult, only a few studies on this topic, mainly related to the dynamic measurement of the steering angle, can be found in the literature [1, 7]. A Datron RV-3 instrument [12] allows measurement of position and orientation of the steering wheel relative to the car body. This measurement, however, is complicated and measured values are not obtained directly but as a result of complex calculations. This instrument has large dimensions and considerable weight as compared to the weight of the wheel. Persistence of the instrument is being reduced under the influence of dynamic loads generated while driving the car over the road unevenness.

2. The goal and scope of the work

The main goal of the work is introduction of an indirect measurement method of the stub axle with steering wheel translation and rotation.

The scope of this work concerns problems of resolving the kinematics of a four-link suspension and the proposed instrument mechanisms, as well as determination of the steering and camber angles and the characteristics of lateral displacements of the wheel centre.

3. Multi-link steering wheels suspension mechanism structure

In Fig. 1, a four-link steering wheel suspension mechanism scheme is shown. Points B_1, B_2, B_4, and B_5 are centres of ball joints connecting links with the stub axle. Point B_3 is the centre of the ball joint connecting steering linkage with the stub axle arm.

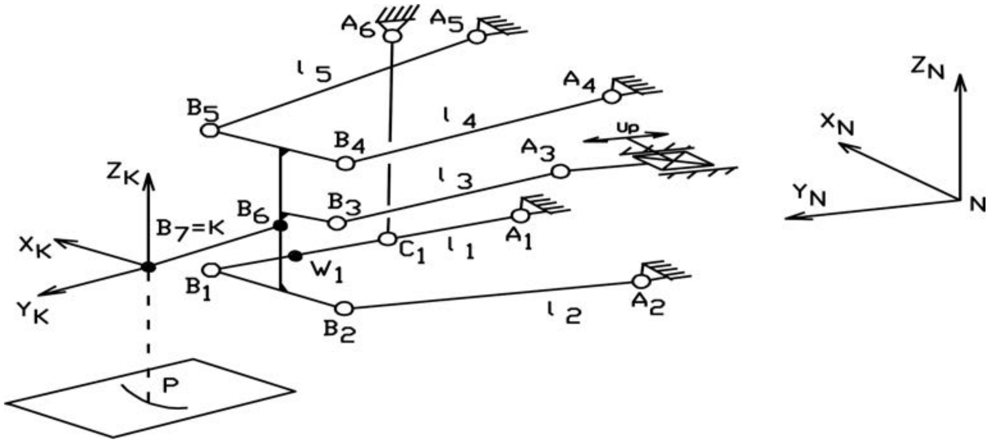

Fig. 1. Scheme of a four-link steering wheels suspension mechanism.

Points A_1, A_2, A_4, and A_5 are centres of ball joints which replace metal-rubber joints connecting links with the car body. Point A_3 is the centre of the ball joint connecting the steering linkage with a rack. A lower front link, represented in the figure by the link A_1B_1, is connected with an anti-roll bar at the point W. A telescopic column is connected with this link at the point C. Frames $\{N\}$ and $\{K\}$ are associated, respectively, with the car body and the stub axle.

4. Equations of geometric constrains of the suspension mechanism

The equations of geometric constraints of the mechanism shown in Fig. 1 can be written as 14 or 5 non-linear algebraic equations. In the first method the equations express squares of distances between characteristic points of the suspension:

$$\begin{aligned} &\mathbf{r}^T_{A_jB_j} \cdot \mathbf{r}_{A_jB_j} = l^2_j, \quad \text{for } j = (1)5, \\ &\mathbf{r}^T_{B_jB_k} \cdot \mathbf{r}_{B_jB_k} = l^2_{jk} \quad \text{for } \begin{cases} j = 1 \text{ and } k = (2)5, \\ j = 2 \text{ and } k = (3)5, \\ j = 3 \text{ and } k = (4)5. \end{cases} \end{aligned} \tag{4.1}$$

In the above system of equations given parameters are: the coordinate z_{B1} of the point $B_1(x_{B1},\, y_{B1},\, z_{B1})$ and the steering rack displacement u_p, added to the coordinate y_{A3} of the point $A_3(x_{A3},\, y_{A3}+u_p,\, z_{A3})$. At the given parameters z_{B1} and u_p, coordinates of the point $B_j(x_{Bj},\, y_{Bj},\, z_{Bk})$, for $j = 1,\ldots,5$ and $k = 2,\ldots,5$ are determined from the system (4.1). The constructional positions of the points B_6 and B_7 are given, therefore, determination of their coordinates in the movements space of the suspension $\{N\}$ is possible from the following systems of equations:

– for the point B_6

$$\mathbf{r}^T_{B_kB_j} \cdot \mathbf{r}_{B_kB_j} = l^2_{kj}, \qquad \text{for} \quad \begin{cases} k = 6 \text{ and } j = 1, \\ k = 6 \text{ and } j = 3, \\ k = 6 \text{ and } j = 5, \end{cases} \tag{4.2}$$

– for the point B_7

$$\mathbf{r}^T_{B_kB_j} \cdot \mathbf{r}_{B_kB_j} = l^2_{kj}, \qquad \text{for} \quad \begin{cases} k = 7 \text{ and } j = 1, \\ k = 7 \text{ and } j = 2, \\ k = 7 \text{ and } j = 4. \end{cases} \tag{4.3}$$

In the second method, the equations of the geometric constraints are expressed by the squares of lengths of vectors beginning and ending, respectively, at the points A_j and B_j, for $j = 1,\ldots,5$, written as:

$$(4.4)\quad (\mathbf{r}_{NK.N}+\mathbf{A}_{NK}\cdot\mathbf{r}_{KB_j.K}-\mathbf{r}_{NA_j.N})^T \cdot(\mathbf{r}_{NK.N}+\mathbf{A}_{NK}\cdot\mathbf{r}_{KB_j.K}-\mathbf{r}_{NA_j.N})=l_j^2, \qquad \text{for} \quad j=1,\ldots,5,$$

where

$$(4.5)\qquad \mathbf{A}_{NK}=\mathbf{A}_\gamma\mathbf{A}_\beta\mathbf{A}_\alpha=\begin{bmatrix} c\gamma & -s\gamma & 0\\ s\gamma & c\gamma & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix} c\beta & 0 & s\beta\\ 0 & 1 & 0\\ -s\beta & 0 & c\beta\end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0\\ 0 & c\alpha & -s\alpha\\ 0 & s\alpha & c\alpha\end{bmatrix}$$

is a rotation matrix of $\{K\}$ against $\{N\}$.

From the system of Eq. (4.4) at given parameters q_3 and u_p, coordinates q_1 and q_2 of the wheel centre $K(q_1, q_2, q_3)$, as well as the rotation angles of $\{K\}$ against $\{N\}$: α, β, γ, are determined.

To ensure equivalence of the computing range of the algorithms based on the systems (4.1) and (4.4), determination of the rotation angles of $\{K\}:\alpha,\ \beta,\ \gamma$ against $\{N\}$ is needed. Thus, the system of Eqs. (4.1) must be supplemented by calculations of the mentioned angles. After calculating the coordinates of the points K and B_j $(j=1,\ldots,5)$ three vectors $\mathbf{r}_{KB_j}$ for $j\in\{1,2,3,4,5\}$ can be created. For each of these vectors a following matrix equation is satisfied:

$$(4.6)\qquad \mathbf{r}_{KB_j.K}=\mathbf{A}_{KN}\cdot\mathbf{r}_{KB_j.N},$$

where $\mathbf{r}_{KB_j.K}$ is a vector in $\{K\}$, $\mathbf{r}_{KB_j.N}$ is a vector in $\{N\}$,

$$(4.7)\qquad \mathbf{A}_{KN}=\mathbf{A}_{NK}^T=\begin{bmatrix} c\beta\cdot c\gamma & c\beta\cdot s\gamma & -s\beta\\ s\alpha\cdot s\beta\cdot c\gamma-c\alpha\cdot s\gamma & s\alpha\cdot s\beta\cdot s\gamma+s\alpha\cdot c\gamma & s\alpha\cdot c\beta\\ c\alpha\cdot s\beta\cdot c\gamma+s\alpha\cdot s\gamma & c\alpha\cdot s\beta\cdot s\gamma-s\alpha\cdot c\gamma & c\alpha\cdot c\beta\end{bmatrix}.$$

Denoting the vectors' coordinates:

$$\mathbf{r}_{KB_j.K}=[x_{bj},y_{bj},z_{bj}]^T, \qquad \mathbf{r}_{KB_j.N}=[x_{jb},y_{jb},z_{jb}]^T$$

and assuming $j=n,m,v$ on the basis of (4.6) we obtain:

$$(4.8)\qquad \begin{aligned} x_{bn}&=(x_{nb}\cdot c\gamma+y_{nb}\cdot s\gamma)c\beta-z_{nb}\cdot s\beta,\\ x_{bm}&=(x_{mb}\cdot c\gamma+y_{mb}\cdot s\gamma)c\beta-z_{mb}\cdot s\beta,\\ x_{bv}&=(x_{vb}\cdot c\gamma+y_{vb}\cdot s\gamma)c\beta-z_{vb}\cdot s\beta.\end{aligned}$$

From the system of Eqs. (4.8), the rotation angles β and γ are calculated. In order to calculate the angle α Eq. (4.9) is used:

$$(4.9)\qquad y_{bv}=(x_{vb}\cdot s\beta\cdot c\gamma)s\alpha-(x_{vb}\cdot s\gamma)c\alpha + (y_{vb}\cdot s\beta\cdot s\gamma)s\alpha+(y_{vb}\cdot c\gamma)c\alpha+(z_{vb}\cdot c\beta)s\alpha.$$

Then, having the rotation angles of the system $\{K\}$ against system $\{N\}$, coordinates of the vector $\mathbf{r}_{B_6B_7.N}$ are:

$$\mathbf{r}_{B_6B_7.N} = \mathbf{A}_{NK} \cdot \mathbf{r}_{B_6B_7.K}, \tag{4.10}$$

the unit vector $\mathbf{e}_K = [e_{kx}, e_{ky}, e_{kz}]^T$, lying on the wheel rotation axis, as well as the steering and camber angles:

$$\delta_k = -\arctan\left(\frac{e_{kx}}{e_{ky}}\right), \tag{4.11}$$

$$\gamma_k = -\text{arc}\ \sin(e_{kz}), \tag{4.12}$$

were calculated.

Calculation of the angles δ_k and γ_k in both algorithms is similar.

It should be noted that solution of the algorithm based on the system of fourteen Eqs. (4.1) takes much less time than the solution based on the system (4.4) consisting of five transcendental equations [10].

5. Solving of the geometric constraints systems of equations of the suspension mechanism

Solutions of the systems of Eqs. (4.1) and (4.4) were obtained by the perturbation method [3, 7]. In the case of the five transcendental Eqs. (4.4) trigonometric functions were expanded into the series:

$$\sin(x_0 + x) = \sin x_0 + x\cos x_0 - \frac{x^2 \sin x_0}{2}, \tag{5.1}$$

$$\cos(x_0 + x) = \cos x_0 - x\sin x_0 - \frac{x^2 \cos x_0}{2}. \tag{5.2}$$

System of equations which can be written down in a general form:

$$f_j(q_1, q_2, \alpha, \beta, \gamma) = 0, \qquad j = 1, \ldots, 5, \tag{5.3}$$

were obtained.

Equations of the system (5.3) were separated into nonlinear and linear parts:

$$f_{jN}(q_1, q_2, \alpha, \beta, \gamma) + f_{jL}(q_1, q_2, \alpha, \beta, \gamma) = 0, \qquad j = 1, \ldots, 5. \tag{5.4}$$

Nonlinear parts of these equations were multiplied by the perturbation parameter ε and a system of auxiliary equations was obtained:

$$g_j(\varepsilon, q_1, q_2, \alpha, \beta, \gamma) = \varepsilon \cdot f_{jN} + f_{jL}, \qquad j = 1, \ldots, 5. \tag{5.5}$$

For $\varepsilon = 1$ systems of Eqs. (5.4) and (5.5) are identical, whereas for $\varepsilon = 0$ system (5.5) consists only of linear equations. It was assumed that solutions of the system (5.5) are the numerical series:

$$
\begin{gathered}
q_1 = \sum_{i=0}^{m} \varepsilon^i q_{1i}, \qquad q_2 = \sum_{i=0}^{m} \varepsilon^i q_{2i}, \\
\alpha = \sum_{i=0}^{m} \varepsilon^i \alpha_i, \qquad \beta = \sum_{i=0}^{m} \varepsilon^i \beta_i, \qquad \gamma = \sum_{i=0}^{m} \varepsilon^i \gamma_i.
\end{gathered} \tag{5.6}
$$

After substituting (5.6) to (5.5) we obtain:

$$
g_j(\varepsilon, q_1(\varepsilon), q_2(\varepsilon), \alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon)) = 0, \qquad j = 1, \ldots, 5. \tag{5.7}
$$

The system of Eq. (5.7) was expanded into a series with respect to ε powers:

$$
\sum_{i=0}^{3} \varepsilon^i g_{ji} = 0, \qquad j = 1, \ldots, 5. \tag{5.8}
$$

Then the linear systems of equations $g_{ji} = 0$ were solved, first for $i = 0$ then for $i = 1, 2, 3$. The solutions which can be presented in a general form:

$$
\begin{gathered}
q_1 = \sum_{i=0}^{3} q_{1i}, \qquad q_2 = \sum_{i=0}^{3} q_{2i}, \\
\alpha = \sum_{i=0}^{3} \alpha_i, \qquad \beta = \sum_{i=0}^{3} \beta_i, \qquad \gamma = \sum_{i=0}^{3} \gamma_i,
\end{gathered} \tag{5.9}
$$

were obtained.

The systems of Eqs. (4.1) were solved in an analogical way.

6. Characteristic points of the suspension mechanism coordinates

A constructional location of the mechanism in the suspension movements space $\{N\}$, defined by the coordinates of ball joints connecting links with the car body and the stub axle [2]:

$A_1\ (144.1, 345.2, -92.2)$; $A_2\ (-229.2, 362.2, -101.7)$;
$A_3\ (-99.7, 400.0, 306.2)$; $A_4\ (-69.0, 396.3, 413.5)$;
$A_5\ (134.6, 428.5, 408.9)$; $B_1\ (28.7, 690.9, -98.0)$;
$B_2\ (-24.4, 687.0, -131.6)$; $B_3\ (-135.7, 617.1, 286.9)$;
$B_4\ (-18.1, 639.8, 388.4)$; $B_5\ (15.4, 673.3, 389.5)$.

The coordinates of points B_6 and $B_7 \equiv K$ lying on the wheel rotation axis:

$$B_6\ (0.5, 647.0, 1.1); \qquad B_7\ (1.0, 747.0, 0.6).$$

Coordinates values were given in [mm].

7. The structure and mobility of the measuring instrument mechanism

The measuring mechanism to determinate the translation and rotation of the steering wheel stub axle shown in Fig. 2 is composed of six links d_i, $i = 1, \ldots, 6$, connected by rotary-sliding kinematic pairs s_i, $i = 1, \ldots, 6$. At the points D_1, D_2, and D_3 links are connected with the stub axle by kinematic pairs. It is characteristic for this mechanism that the point D_1 is a common centre of three joints, point D_2 is a centre of two such joints, and the point D_3 is a centre of the ball joint. By points H_i, $i = 1, \ldots, 6$, centres of ball joints connecting links with the car body were noted in the figure.

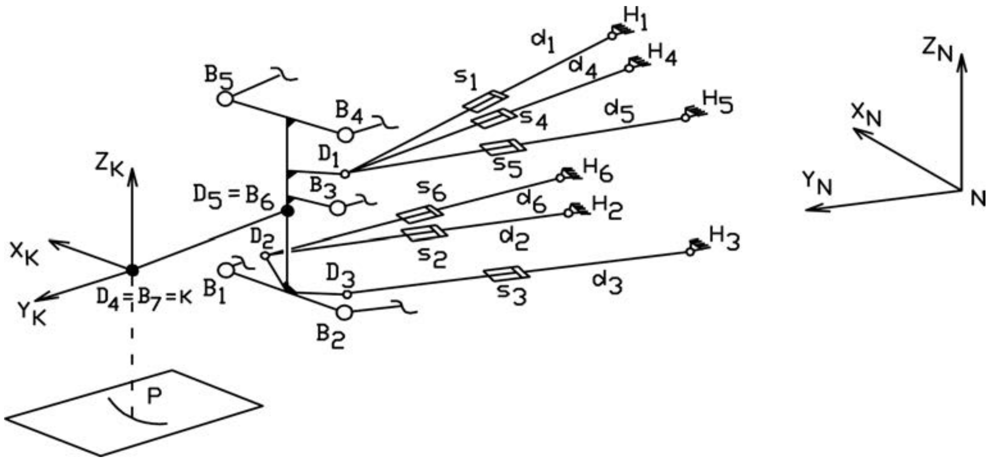

Fig. 2. Scheme of the measuring instrument to determine the translation and rotation of the steering wheel stub axle.

The instrument mechanism has 6 mobility degrees – using the formula from the theory of mechanisms and machines:

$$R = R_t - R_p, \tag{7.1}$$

where

$$R_t = 6(n-1) - \sum_{i=1}^{5} i \cdot p_i, \tag{7.2}$$

where R is real mobility of the mechanism, R_t is theoretical mobility of the mechanism, R_p is apparent mobility of the mechanism, p_i is kinematic pairs of the i-th class, n is number of links creating the mechanism.

With regard to the considered mechanism: $n = 13$, $p_4 = 6$, $p_3 = 12$, $p_5 = p_2 = p_1 = 0$, $R_t = 18$. After subtracting the apparent degrees of mobility $R_p = 12$ from the theoretical mobility, the real mobility $R = 6$; it is equal to the stub axle degrees of freedom in {N}.

8. Kinematics of the measuring instrument mechanism

Centres of the ball joints: D_1, D_2, and D_3 belong to the wheel stub axle. So the distances of these points from the points B_j, $j = 1,\ldots,6$ can be calculated. However, the coordinates of the points: D_1, D_2, and D_3 in $\{N\}$ can be determined from the systems of equations:

– for the point D_1

$$\mathrm{r}^T_{B_jD_1.N} \cdot \mathbf{r}_{B_jD_1.N} = l^2_{D_1B_j}, \qquad \text{for} \quad j = 1,2,3. \tag{8.1}$$

– for the point D_2

$$\mathbf{r}^T_{B_jD_2.N} \cdot \mathbf{r}_{B_jD_2.N} = l^2_{D_2B_j}, \qquad \text{for} \quad j = 1,3,5. \tag{8.2}$$

– for the point D_3

$$\mathbf{r}^T_{B_jD_{3.N}} \cdot \mathbf{r}_{B_jD_3.N} = l^2_{D_3B_j}, \qquad \text{for} \quad j = 2,4,5. \tag{8.3}$$

Thus, it becomes possible to calculate relative elongations s_i of links d_i, $i = 1,\ldots,6$, relative to their constructional distances. Elongations s_i depend on the given parameters q_3 and u_p, i.e., $s_i(q_3,\, u_p)$.

In practical applications of the measuring instrument for determining the rotation and translation of the steering wheel stub axle it is needed to measure the coordinates of points D_1, D_2, and D_3, as well as H_i, $i = 1,\ldots,6$, for the constructional suspension configuration; links elongations $s_i(q_3,\, u_p)$ measured by the sensors are also needed.

Determination of the steering wheel stub axle rotation and translation using a measuring instrument boils down to solving of the inverse problem. In this case, at given elongations $s_i(q_3$ and $u_p)$ of the links d_i, $i = (1)6$ and constructional positions of the points D_1, D_2, and D_3, as well as the additional point, e.g., $D_4 \equiv B_7$, the coordinates of these points as the functions of parameters q_3 and u_p should be determined from the systems of equations:

– for the point D_1

$$\mathbf{r}^T_{D_1H_i.N} \cdot \mathbf{r}_{D_1H.N} = (l_{D_1H_i} + s_i)^2, \qquad \text{for} \quad i = 1,4,5. \tag{8.4}$$

– for the point D_2

$$\begin{aligned} &\mathbf{r}^T_{D_2D_1.N} \cdot \mathbf{r}_{D_2D_1.N} = l^2_{D_2D_1}, \\ &\mathbf{r}^T_{D_2H_i.N} \cdot \mathbf{r}_{D_2H_i.N} = (l_{D_2H_i} + s_i)^2, \qquad \text{for} \quad i = 2, 6. \end{aligned} \tag{8.5}$$

– for the point D_3

$$\begin{aligned} &\mathbf{r}^T_{D_3D_1.N} \cdot \mathbf{r}_{D_3D_1.N} = l^2_{D_3D_1}, \\ &\mathbf{r}^T_{D_3D_2.N} \cdot \mathbf{r}_{D_3D_2.N} = l^2_{D_3D_2}, \\ &\mathbf{r}^T_{D_3H_3.N} \cdot \mathbf{r}_{D_3H_3.N} = (l_{D_3H_3} + s_3)^2. \end{aligned} \tag{8.6}$$

– for the point D_4

$$\begin{aligned} &\mathbf{r}^T_{D_4D_1.N} \cdot \mathbf{r}_{D_4D_1.N} = l^2_{D_4D_1}, \\ &\mathbf{r}^T_{D_4D_2.N} \cdot \mathbf{r}_{D_4D_2.N} = l^2_{D_4D_2}, \\ &\mathbf{r}^T_{D_4D_3.N} \cdot \mathbf{r}_{D_4D_3.N} = l^2_{D_4D_3}. \end{aligned} \tag{8.7}$$

The coordinates of the point $\mathrm{D}_5 \equiv \mathrm{B}_6$ are calculated similarly to the one of the point D_4.

The way of determining the angles of rotation α, β, γ $\{K\}$ against $\{N\}$, the steering and camber angles δ_d, γ_d on the basis of the coordinates of the points D_i, $i = 1, \ldots, 5$, and the points B_j, $j = 1, \ldots, 6$, is the same.

9. Coordinates of the measuring instrument to the car body and the stub axle anchorage points

The coordinates of the centres of joints D_1, D_2, and D_3 in [mm], as well as coordinates of the points D_4 and D_5 belonging to the wheel rotation axis assigned to the constructional configuration of the suspension are given below:

$$\begin{aligned} &H_1(90.0, 355.0, -40.0); \quad &&H_2(10.0, 250.0, -70.0); \quad &&H_3(-40.0, 280.0, 270.0); \\ &H_4(45.0, 360.0, -90.0); &&H_5(110.0, 362.0, 10.0); &&H_6(30.0, 300.0, -95.0); \\ &D_1(50.0, 560.0, -30.0); &&D_2(25.0, 580.0, -90.0); &&D_3(-50.0, 600.0, 200.0). \end{aligned}$$

The coordinates of the points $D_4 \equiv K$ and D_5 belonging to the wheel rotation axis are as follows:

$$D_4(1.0, 747.0, 0.6); \qquad D_5(0.5, 647.0, 1.1).$$

10. Characteristics of the stub axle rotation angles

In Fig. 3 the stub axle rotation angles characteristics are shown. α, β, and γ are angles of rotation of the frame $\{K\}$ against the x, y, z axes of the frame $\{N\}$.

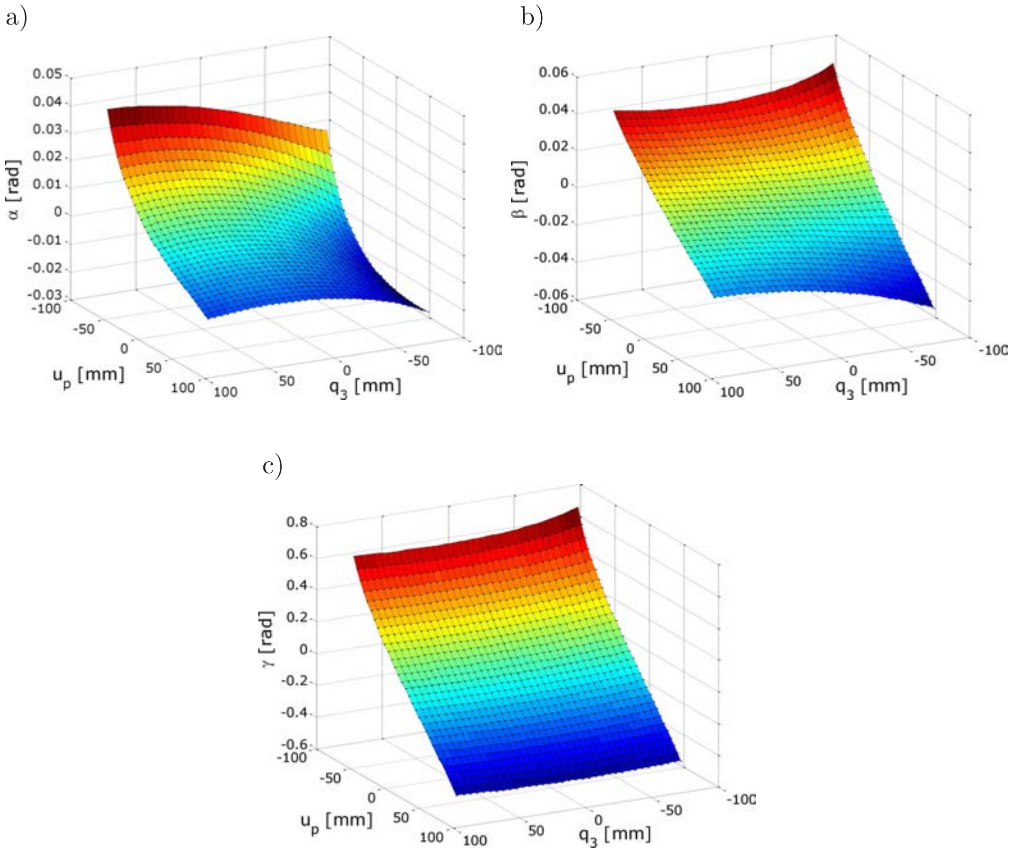

Fig. 3. Dependences of the stub axle rotation angles on suspension deflection q_3 and steering rack displacement u_p.

11. Elongations of the measuring instrument links as a function of the suspension deflection and the steering rack displacement

The following graphs show dependences of individual links of the measuring instrument elongations on the steering rack displacement u_p and the suspension deflection q_3. As the elongations of the sensors are simultaneously the suspension deflection and the steering rack displacement functions, they were shown on spatial graphs.

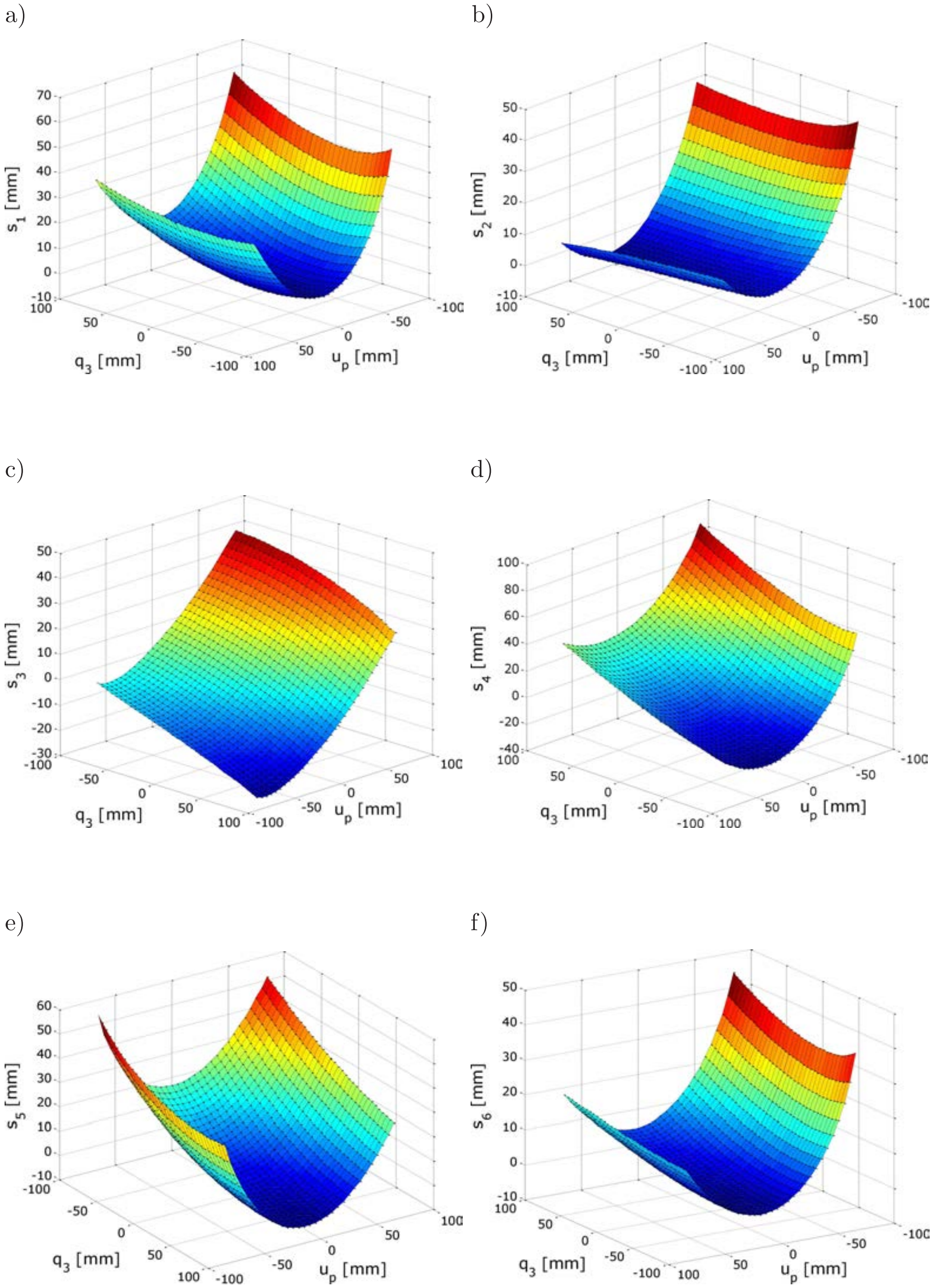

FIG. 4. Dependences of the measuring instrument links elongations on suspension deflection q_3 and steering rack displacement u_p.

12. Suspension characteristics

In Figs. 5, 6, and 7 characteristics of the analysed mechanism obtained on the basis of its kinematics solution and using the described measuring instrument are shown.

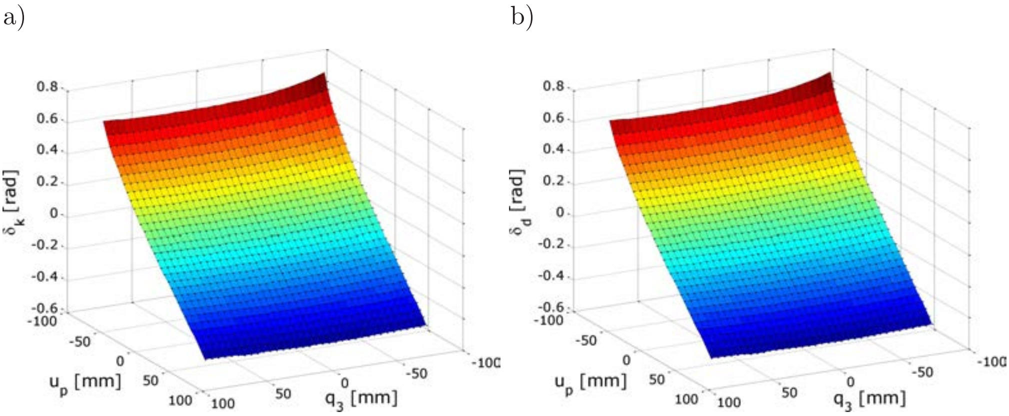

FIG. 5. Dependences of the steering angle δ on suspension deflection q_3 and steering rack displacement u_p, determined on the basis of: a) the suspension kinematics solution, b) using measuring instrument.

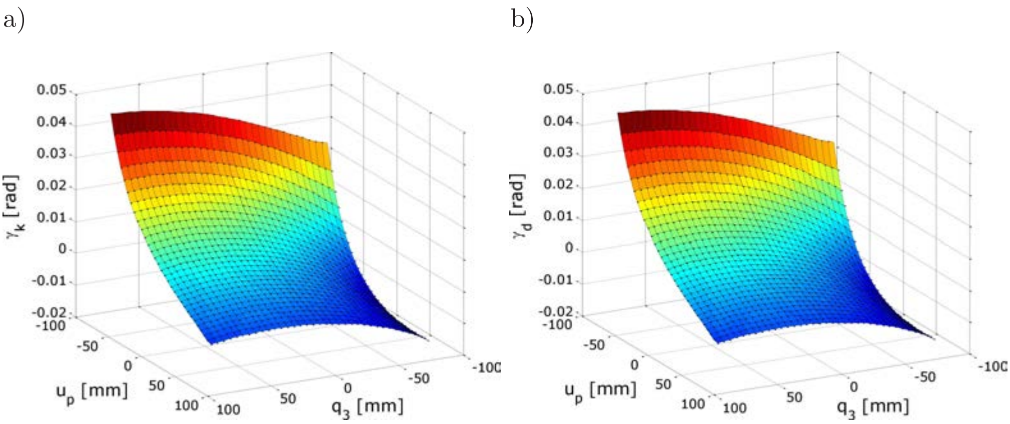

FIG. 6. Dependences of the camber angle γ on suspension deflection q_3 and steering rack displacement u_p, determined on the basis of: a) the suspension kinematics solution, b) using measuring instrument.

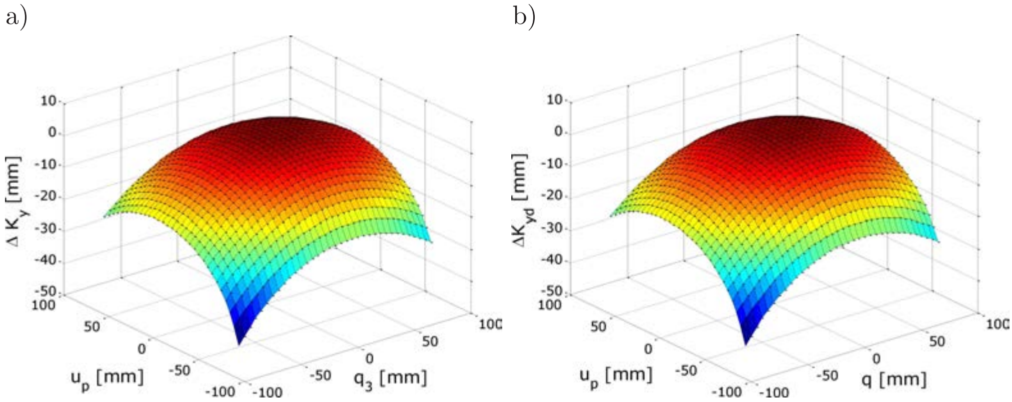

Fig. 7. Relative changes in the lateral position of the wheel centre, depending on suspension deflection q_3 and steering rack displacement u_p, determined on the basis of: a) the suspension kinematics solution, b) using measuring instrument.

13. Conclusion

The structure of the proposed measuring instrument mechanism enables its connection to the stub axle via three joints. One of these joints is a conjunction of three joints with a common centre, each with three degrees of freedom. This solution allows easiest measuring instrument joints to the stub axle connection. A Stewart platform mechanism can be used instead of the presented instrument to translation and rotation of the stub axle determination; it requires six ball joints connections to the stub axle.

The basis of the method of indirect measurement of translation and rotation of the stub axle with the steering wheel in the suspension movements space are algorithms that include geometric constraints system of equations of four-link suspension and measuring instrument mechanisms. Solutions of these systems of equations were used to compile characteristics of steering and camber angles and of wheel centre displacement in the lateral direction.

On the basis of an analysis of the results of a computer simulation of the measure method it is to conclude that the worked out instrument can be used in experimental car tests.

Analysis of the kinematic characteristics contained in the work shows that the examined mechanism does not have singular points in the suspension movements space.

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Received June 10, 2011; revised version January 12, 2012.