RUMBLING IN HIGH SPEED PASSENGER TRAINS

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A model of a flexible wheel set running on flexible rails is presented. It gives hints for the reasons of rumbling in the high-speed passenger train InterCityExpress (ICE) of the German railway company (DB) if these trains travel on the new stiff tracks. The rumbling results from non-circular wheels which cause oscillations of the whole system. The problem of polygonization of railway wheels is not considered.

1. INTRODUCTION

The 100 Hz-rumbling of the German high-speed train ICE in the passenger coaches diminishes the travel comfort. An investigation of the excitation mechanism is necessary to understand the problem and to avoid this noise. The characteristics of the rumbling are as follows:

1. The frequency of the rumbling is in the range of 90 Hz to 110 Hz, nearly independent of the train velocity.
2. The rumbling spectrum shows higher harmonics of the revolutions (or the mean angular velocity \( \Omega \)) of the wheel sets.
3. The rumbling noise increases with increasing non-circularity of the wheels.
4. The rumbling occurs mainly on the new, stiff tracks of DB.

2. THE MODEL

We assume that the non-circular profiles of the wheels are periodic functions of \( \varphi \), the azimuthal variable. These non-circularities produce oscillations of the whole system consisting of the coach and the track. We consider here only a part of the system which is sketched in Fig. 1. It consists of the rails and one wheel set which is guided by the wheel frame (since in our model the wheel frame can not rotate, we prefer this name instead of bogie). The wheel frame moves with constant velocity \( v \) in \( e_{g1} \)-direction. The wheel frame and the wheel set are joined by three spring-damper pairs (not sketched in Fig. 1) at each end of the axle.
The rails, the rims of the wheels, and the axle are described in the model as one-dimensional continua; the wheel disks by two-dimensional continua. We call a flexible body an $n$-dimensional continuum ($n$ being a natural number) if the deflection(s) depend(s) on $n$ independent spatial variables. The hubs are assumed to be rigid bodies. We assume a linear viscoelastic law for the stress-strain relation, i.e. $\sigma_{ij} = E_{ij}^{kl} \left(1 + \eta \partial / \partial t \right) \varepsilon_{kl}$ where $E_{ij}^{kl}$ is the tensor of elasticity and $\eta$ is the so-called relaxation time (cf. Fung [1], p. 24).

![Fig. 1. The model.](image)

The deflections $u_{wa}$ and $w_{wa}$ of the axle and the angles of rotation $\alpha_{wa}$, $\gamma_{wa}$ of the cross-section in $\varepsilon_{g1}$- and $\varepsilon_{g3}$- direction (cf. Fig. 2, $\varepsilon_{g1} = \varepsilon_{wa1}$, $\varepsilon_{g3} = \varepsilon_{wa3}$), respectively, are governed by two systems of partial differential equations of Timoshenko beams. Due to the rotation of the axle, the two systems are coupled (cf. Wauer [11]). For the deflection $v_{wa}$ and the angle of rotation $\beta_{wa}$ of the axle we choose the partial differential equations of a bar subject to extension and torsion (cf. Pilkey [8], p. 593). The functions $u_{wa}$, $v_{wa}$, $w_{wa}$, $\alpha_{wa}$, $\beta_{wa}$, and $\gamma_{wa}$ depend on the spatial coordinate $y$ in the $\varepsilon_{wa2}$-direction and the time $t$.

We describe the degrees of freedom for the wheels and the rails, e.g. for wheel 1 and rail 1 and drop the index 1. The hub has six degrees of freedom: three for translation $u_{wh}$, $v_{wh}$, $w_{wh}$ and three for rotation $\alpha_{wh}$, $\beta_{wh}$ $\gamma_{wh}$ (cf. Fig. 3). The deflections of the hubs and those of the axle at its ends are coupled by geometrical and mechanical boundary conditions.
FIG. 2. Deflections of the axle.

FIG. 3. Degrees of freedom of hub 1.

FIG. 4. Deflections of disk 1.
We assume that the wheel disk is clamped at the hub. Kirchhoff's plate theory governs the deflection $v_{wd}$ of the disk perpendicular to its middle plane (cf. Fig. 4). The deflections $u_{wdr}$, $u_{wdt}$ in the middle plane are governed by the equations of a shell (cf. Washizu [10], p. 159). The functions $v_{wd}$, $u_{wdr}$, $u_{wdt}$ depend on the azimuthal angle $\varphi$, the radial coordinate $r$, and time $t$.

The partial differential equations of a Timoshenko beam and a bar in torsion and extension describe the motions of the rim. There is one additional degree of freedom for each rim. This is the rotation $\beta_{wr}$ of the cross-sections with respect to the $e_{wh2}$ direction ($e_{wh}$ is the local coordinate system fixed to the hub). The equations of motion of the rim are obtained by following the way outlined by Teichmann [9].

The deflections $v_r$ and $w_r$ of the rail in $e_{g2}$- and $e_{g3}$-direction, respectively, obey the partial differential equations of a Timoshenko beam. The corresponding angles are $\gamma_r$ and $\beta_r$ (cf. Fig. 5). The deflections $u_r$ and the rotation $\alpha_r$ are governed by the equations of a bar in extension and torsion, respectively. The deflections $u_r$, $v_r$, $w_r$ and the angles $\alpha_r$, $\beta_r$, $\gamma_r$ depend on the spatial coordinate $s := x - vt - u_{cr}$ (moving coordinate system; the point of contact is located on the cross-section $Q_{cr}$ whose position is given by $u_{cr}$, cf. Fig. 5), and on time $t$.

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**Fig. 5.** Deflections of rail 1.
The wheels and the respective rails are coupled via Kalker’s linear contact theory (cf. KALKER [2]), the linearized Hertzian contact stiffnesses, geometrical and mechanical boundary conditions, and equations which describe the smoothness of the wheels and the rails at the points of contact. We call these equations the contact equations.

The resulting set of equations is linear, homogeneous, autonomous and it consists of partial and ordinary differential equations and algebraic equations.

The equations of motion are derived by the principle of Hamilton-Ostrogradskij:

\[
0 = \delta S = \int_{t=t_1}^{t_2} (\delta T - \delta U + \tilde{\delta}W) \, dt .
\]

Here \(\delta T\) is the variation of the kinetic energy, \(\delta U\) is the variation of the potential energy, and \(\tilde{\delta}W\) are the virtual works of forces, which can not be derived by gradient calculation from a potential.

For the \(r\)-dependence of the disk deflections \(v_{wd}, u_{wdr}, u_{wdt}\) we choose the following shape function, i.e. polynomials in \(r\) (\(R_{wdi}\) is the inner radius of the wheel disk):

\[
u_{wdt}(r, \varphi, t) = a_{t1}(\varphi, t)(r/R_{wdi} - 1) + a_{t2}(\varphi, t)(r/R_{wdi} - 1)^2 ,
\]

\[
u_{wdr}(r, \varphi, t) = a_{r1}(\varphi, t)(r/R_{wdi} - 1) + a_{r2}(\varphi, t)(r/R_{wdi} - 1)^2 ,
\]

\[
u_{wd}(r, \varphi, t) = a_{v1}(\varphi, t)(r/R_{wdi} - 1)^2 + a_{v2}(\varphi, t)(r/R_{wdi} - 1)^3 .
\]

Since the disk is clamped at the hub, the shape function for \(v_{wd}\) has no constant or linear terms in \((r/R_{wdi} - 1)\).

Putting these functions in (2.1) one obtains, after partial integration with respect to time and the space variables, the partial differential equations for the continuous parts of the system, the ordinary differential equations for the hub (rigid body) and the mechanical boundary conditions. After transformation of the material coordinate \(\varphi\) to \(\psi := \varphi - \Omega t + \beta_{wh} - \beta_{wc}\) (\(v_{wd} = (a_{v1}, a_{v2})^T\), \(u_{wd} = (a_{r1}, a_{r2}, a_{t1}, a_{t2}, \beta_{wr})\), \(\beta_{wc}\) determines the point of contact on the rim), the partial differential equations for the wheel disk and the rim are:

\[
0 = D_{wp}[v_{wd}] + g_{wpv} \ddot{v}_{wh} + \sin(\psi)(\dot{a}_{wh}g_{wp\alpha1} + \dot{\gamma}_{wh}g_{wp\gamma2})
+ \cos(\psi)(\dot{a}_{wh}g_{wp\alpha2} + \dot{\gamma}_{wh}g_{wp\gamma1}) ,
\]

\[
0 = D_{ws}[u_{wd}] + g_{ws\beta1}\dot{\beta}_{wh} + g_{ws\beta2}\dot{\beta}_{wh} + \sin(\psi)(\dot{\dot{u}}_{wh}g_{wsu1}
+ \ddot{u}_{wh}g_{wsu2} + \dot{\dot{u}}_{wh}g_{wsu1} + \dot{\dot{u}}_{wh}g_{wsu2}) + \cos(\psi)(\dot{\dot{u}}_{wh}g_{wsu3}
+ \ddot{u}_{wh}g_{wsu4} + \dot{\dot{u}}_{wh}g_{wsu3} + \dot{\dot{u}}_{wh}g_{wsu4}) .
\]
Here $D_{wp}$ and $D_{ws}$ are linear differential operators (differentiation with respect to $\psi$ and $t$). With respect to $\psi$, $D_{wp}$ is of fifth order and $D_{ws}$ is of third order, with respect to $t$ they are of second order. The order with respect to $\psi$ is by one higher than the order of the Kirchhoff plate theory or the shell theory, respectively, because material damping and the moving coordinate $\psi$ raises the order (cf. MEYWERK [3]). The tuples $g_{wp}$ have five elements, the tuples $g_{ws}$ have two elements. The coupling terms between the hub deflections and rotations on the one side and plate and shell deflections on the other side, result from the terms in the kinetic energy. Thus, these tuples depend on the geometry and on the mass densities. Elastic constants occur in $D_{wp}$ and $D_{ws}$ only.

We restrict the linearized ordinary differential equations for the hub to those for $v_{wh}$, $\alpha_{wh}$, and $u_{wh}$. The remaining three equations look similar:

\begin{align}
0 &= m_w \ddot{v}_{wh} + b_{whv} \dot{v}_{wh} + k_{whv} v_{wh} \\
&\quad + \int_{\psi=0}^{2\pi} g^T_{whv}(\ddot{v}_{wd} - 2\Omega\dot{v}'_{wd} + \Omega^2 v''_{wd}) \, d\psi + F_{c2} + F_{wa2}, \\
0 &= J_{w1} \ddot{\alpha}_{wh} + \Omega J_{w2} \dot{\gamma}_{wh} + b_{whw}(b^2_{ws} \dot{\alpha}_{wh} + b_{ws} \ddot{w}_{wh}) \\
&\quad + k_{whw}(b^2_{ws} \alpha_{wh} + b_{ws} w_{wh}) + \int_{\psi=0}^{2\pi} \left\{ \sin(\psi)[g^T_{whw1}(\dot{v}_{wd} - \Omega v'_{wd}) \\
&\quad + g^T_{whw2}(\ddot{v}_{wd} - \Omega \dot{v}''_{wd} + v'''_{wd}) + g^T_{whw3} v'_{wd} + \cos(\psi)[g^T_{whw4} v_{wd} \\
&\quad + g^T_{whw5}(\ddot{v}_{wd} - \Omega \dot{v}'_{wd} + v''_{wd}) + g^T_{whw6}(\dot{v}_{wd} - \Omega \dot{v}_w + v''_{wd})] \right\} \, d\psi + F_{c2}(R_{wdo} \\
&\quad + \tilde{h}_{wh}(0)) + M_{wa1} + P(-\alpha_{wh}(R_{wdo} + \tilde{h}_{wh}(0)) + v_{cw} + g_{c}(v_{wd})], \\
0 &= m_w \ddot{u}_{wh} + b_{whu}(\dot{u}_{wh} - b_{ws} \gamma_{wh}) + k_{whu}(u_{wh} - b_{ws} \dot{\gamma}_{wh}) \\
&\quad + F_{c1} + F_{wa1} + \int_{\psi=0}^{2\pi} \left\{ \sin(\psi)[g^T_{whu1} u_{wd} + g^T_{whu2}(\dot{u}_{wd} - \Omega u'_{wd}) \\
&\quad + g^T_{whu3}(\ddot{u}_{wd} - \Omega \dot{u}''_{wd} + u'''_{wd}) + \cos(\psi)[g^T_{whu4} u_{wd} \\
&\quad + g^T_{whu5}(\ddot{u}_{wd} - \Omega \dot{u}'_{wd} + u''_{wd}) + g^T_{whu6}(\dot{u}_{wd} - \Omega \dot{u}_w + u''_{wd})] \right\} \, d\psi.
\end{align}

Here $m_w$ is the mass of the whole wheel, $b_{wh}$, $k_{wh}$ (\(\cdot\) stands for $u$, $v$, $w$) are the damping or stiffness, resp., of the primary suspension of the wheel, i.e. the
connection between the hub and the wheel frame. The moments of inertia are \( J_{wi} (i = 1, 2) \) and the distance between the centre of mass of the hub and the primary suspension is \( b_{ws} \). The contact forces are denoted by \( F_{ci} \) and the forces and moments between the axle and the hub by \( F_{wa1} (i = 1, 2, 3) \) and \( M_{wa1} \). The tuples \( g_{wh} \) are due to the coupling between the hub and the disk. Furthermore, \( R_{wds} \) is the outer radius of the wheel disk, \( \tilde{h}_{wr}(\psi) \) is the height of the rim in dependence on the coordinate \( \psi \). This means, that \( \tilde{h}_{wr}(0) \) is the height of the rim at the point of contact. The static load \( P \) represents the weight of the coach, \( g_c \) is a linear differential operator, and \( v_{cw} \) is the displacement of the point of contact in the \( \vec{e}_2 \)-direction. The expression with the load \( P \) takes into account the moment which acts if the point of contact does not lie in the direction of \( P \).

The mechanical boundary conditions are:

\[
(0, 0)^T = [B_{wp1}[v_{wd}]_0^{2\pi_+} + F_{c2}b_{wp1}^T + Pb_{wp2}(v_{wd})
\]

\[
(0, 0)^T = [B_{wp2}[v_{wd}]_0^{2\pi_+} + Mcb_{wp3},
\]

\[
(0, 0, 0, 0, 0)^T = [B_{wp3}[u_{wd}]_0^{2\pi_+} + F_{c1}b_{wp4}^T + F_{c3}b_{wp5}^T + Pb_{wp6}(u_{wd}),
\]

\[
(0, 0)^T = [v_{wd}]_0^{2\pi_-},
\]

\[
(0, 0)^T = [u_{wd}]_0^{2\pi_+}.
\]

Here \( B_{wp1}, B_{wp2}, \) and \( B_{wp3} \) are linear differential operators with respect to \( \psi \) and \( t \) (the time derivatives in the boundary conditions are due to the moving coordinate \( \psi \) and the material damping). With respect to \( \psi \), they have the order of four \( (B_{wp1}), \) three \( (B_{wp2}), \) and two \( (B_{wp3}), \) and with respect to \( t \) they have the order of one. The tuples \( b_{wp i} (i = 1, \ldots, 6) \) depend on the geometry. Furthermore, \( b_{wp1}, b_{wp2}, b_{wp4}, \) and \( b_{wp5} \) depend on \( \tilde{h}_{wr}(0) \) and \( b_{wp2} \) and \( b_{wp6} \) are a linear function of \( v_{wd} \) and \( u_{wd}, \) respectively. The notation \([f]_0^{2\pi_+}\) stands for \([f]_0^{2\pi_-} := \lim_{\psi \to 2\pi} f(\psi) - \lim_{\psi \to 0} f(\psi)\).

The partial differential equations which govern the motion of the axle have been found by Wauer [11]:

\[
(0, 0, 0, 0) = D_{wa1}[u_{wa1}, w_{wa1}, \alpha_{wa}, \gamma_{wa}],
\]

\[
0 = D_{wa2}[v_{wa}],
\]

\[
0 = D_{wa3}[\beta_{wa}].
\]

Here \( D_{wa1}, D_{wa2}, \) and \( D_{wa3} \) are linear partial differential operators with respect to \( y \) (of second order) and \( t \) (second order). The partial differential equations
(Timoshenko beam) for \( u_{wa}, \gamma_{wa} \) on one side and \( w_{wa}, \alpha_{wa} \) on the other side, are coupled since the axle rotates; they would be decoupled for a vanishing angular velocity of the axle. Furthermore, the motions of the axle and the hubs are coupled by geometrical conditions. We give e.g. two coupling equations (for wheel 1), the other ten look similar (\( \ell \) is the length of the axle and \( b_{wh} \) is the height of the hub in \( \vec{e}_{g2} \)-direction):

\[
0 = u_{wa}(\ell) - u_{wh} + \gamma_{wh} b_{wh}/2, \quad 0 = \gamma_{wa}(\ell) - \gamma_{wh}.
\]

(2.18)

For the shearing forces and moments \( F_{wai} \) and \( M_{wai} \) (i = 1, 2, 3) we give only two examples (for wheel 1), the other ten look again similar or are well known (\( EA_{wa} \) is the stiffness in extension and \( EI_{wa} \) in bending, \( \eta \) is the material damping parameter):

\[
0 = F_{wa2} - EA_{wa}(\nu'_{wa}(\ell) + \eta \nu''_{wa}(\ell)),
\]

\[
0 = M_{wa1} - EI_{wa}(\alpha'_{wa}(\ell) + \eta \alpha''_{wa}(\ell)),
\]

(2.19)

The partial differential equations for the rail read:

\[
0 = D_{r1}[(w_r, \beta_r, w_r)],
\]

(2.20)

\[
0 = D_{r2}[(u_r, \alpha_r, \gamma_r)].
\]

(2.21)

Here \( D_{r1} \) and \( D_{r2} \) are linear differential operators with respect to \( s \) (of order three) and \( t \) (of order two). The movement of the coordinate \( s = x - vt - u_{rc} \) and the material damping are the reasons for raising the order with respect to \( s \); the reader would have expected order two because of the Timoshenko beam and the bar in torsion and extension. The partial differential equations for \( u_r \) on the one side and \( \beta_r, w_r \) on the other side are coupled because the Winkler foundation acts on the rail foot and because the centre of shear and the centre of mass of the rail cross-sections do not coincide. The same holds for the partial differential equation for \( \alpha_r \) (torsion) and \( v_r, \gamma_r \) (Timoshenko beam). The mechanical boundary conditions at the point of contact are:

\[
0 = F_{c1} + [B_{r1}[u_r]]_{0^+}^{0^-},
\]

(2.22)

\[
0 = F_{c3} + [B_{r3}[w_r, \beta_r]]_{0^+}^{0^-},
\]

\[
0 = F_{c1} g_{r3} + P g_{r4}(\beta_r) + [B_{r5}[\beta_r]]_{0^+}^{0^-},
\]

\[
0 = [u_r]_{0^+}^{0^-},
\]

\[
0 = [w_r]_{0^+}^{0^-}.
\]
\[ (2.22) \quad 0 = [\beta_r]_0^{-}, \]
\[ 0 = F_{c2} + [B_r2[v_r, \gamma_r]]_0^{-}, \]
\[ 0 = F_{c2}g_{r1} + P_{r2}(\alpha_r) + [B_r4[\alpha_r]]_0^{-}, \]
\[ 0 = M_c + [B_r6[\gamma_r]]_0^{-}, \]
\[ 0 = [v_r]_0^{-}, \]
\[ 0 = [\alpha_r]_0^{-}, \]
\[ 0 = [\gamma_r]_0^{-}. \]

Here \( B_{ri} \) (\( i = 1, \ldots, 6 \)) are linear differential operators with respect to \( s \) (order two) and \( t \) (order one) and \( g_{ri} \) (\( i = 1, \ldots, 4 \)) depend on geometric parameters and stiffnesses.

The contact equations can be summarized in the following form:

\[ (2.23) \quad (0, 0, F_{c3})^T = g_{cr1}(u_r, v_r, w_r, \alpha_r, \beta_r, \gamma_r)|_{s=0} \]
\[ -g_{cw1}(\tilde{h}_{wr}, u_{wh}, v_{wh}, w_{wh}, \alpha_{wh}, \beta_{wh}, \gamma_{wh}, v_{wd}, u_{wd})|_{\psi=0}, \]

\[ (2.24) \quad (0, 0)^T = D_{cr1}[u_r, v_r, w_r, \alpha_r, \beta_r, \gamma_r]|_{s=0} - D_{cw1}[\partial \tilde{h}_{wr}/\partial \psi, \]
\[ m u_{wh}, v_{wh}, w_{wh}, \alpha_{wh}, \beta_{wh}, \gamma_{wh}, v_{wd}, u_{wd}]|_{\psi=0}, \]

\[ (2.25) \quad (\nu_1, \nu_2, \Phi_c)^T = D_{cr2}[u_r, v_r, w_r, \alpha_r, \beta_r, \gamma_r]|_{s=0} \]
\[ - D_{cw2}[u_{wh}, v_{wh}, w_{wh}, \alpha_{wh}, \beta_{wh}, \gamma_{wh}, v_{wd}, u_{wd}]|_{\psi=0}, \]

\[ (2.26) \quad \begin{pmatrix} F_{c1} \\ F_{c2} \\ M_c \end{pmatrix} = G a_{cc} b_{ce} \begin{pmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & \sqrt{a_{cc} b_{ce} c_{23}} \\ 0 & -\sqrt{a_{cc} b_{ce} c_{23}} & a_{cc} b_{ce} c_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \Phi_c \end{pmatrix}. \]

Here \( g_{cr1} \) and \( g_{cw1} \) are linear functions. The first two equations of (2.23) ensure that the wheel and the rail do not loose contact. The third equation of (2.23) is the linearized Hertzian contact equation. The operators \( D_{cr1} \) and \( D_{cw1} \) are linear differential operators with respect to \( s \) and \( \psi \), respectively. Equations (2.24) ensure that the tangential planes of the wheel and the rail in the point of contact are parallel. The operators \( D_{cr2} \) and \( D_{cw2} \) are linear differential operators with respect to \( s \) and \( \psi \) (order one), respectively, and with respect to \( t \) (order one). Here \( \nu_1 \) and \( \nu_2 \) are the slips, i.e. the relative velocities in the point of contact between the wheel and rail divided by \( v \), and \( \Phi_c \) is the spin. These quantities enter
the equation of Kalker's linear theory (2.26). In (2.26) \( G \) is the shear modulus, \( a_{ce} \)
and \( b_{ce} \) are the radii of the contact ellipse, and \( c_{11}, c_{22}, c_{23}, \) and \( c_{33} \) are Kalker's
coefficients.

From Hertz's and Kalker's theory it is known, that the constants \( a_{ce}, b_{ce} \) and
Kalker's coefficients in (2.26) fluctuate, i.e. they are not constant. For example,
the radii of the contact ellipse depend on the normal force and on the mutual
wheel rail position. Furthermore, Kalker's coefficients depend on these param-
ters, too. We assume in our approach first, that the rail surface is an ideal cylinder
and the wheel surface is an ideal cone; second, that the changes of the mutual
wheel-rail positions are small, i.e. we consider small oscillations only; and third,
that the fluctuations in the normal force are small in comparison with the static
load \( P \approx 60 \text{kN} \), which is the mean value of the normal force. Assuming this,
we can neglect the fluctuation in the radii and in Kalker's coefficients.

On the one hand, the model of the wheel set and the rail is very accurate but
on the other hand, the foundation of the rail is taken into account by a uniform
Winkler foundation neglecting the periodic character. In the author's opinion,
there are two reasons for the fact that the periodic foundation do not influence
the rumbling noise:

1. If the periodic foundation would cause the rumbling noise, then the noise
must occur frequently. But it is known from experience that noise in the ICE
trains occurs if the wheels are out-of-round and the noise vanishes if the wheels
are round.

2. If the periodic foundation would cause the rumbling, there must be a peak
in the spectrum which has to fit the periodic excitation of the foundation. In
MÜLLER-BORUTTAU and EBERSBACH [7] the spectrum of the measured rumbling
noise in an ICE train (velocity 260 km/h \( \approx 72.22 \text{ m/s} \), radius of the wheels \( R =
0.46 \text{ m} \), sleeper distance 0.6 m) is published. In the frequency range from 20 Hz
to 130 Hz there occur six peaks: 25 Hz (95 dB), 100 Hz (95 dB), 50 Hz (92 dB),
75 Hz (87 dB), 125 Hz (82 dB), 120 Hz (80 dB). The values in brackets denote the
heights of the peaks. The first five peaks result from the out-of-round wheel (the
frequencies are multiples of \( (72.22 \text{ m/s})/(2\pi 0.46 \text{ m}) \approx 24.99 \text{ Hz} \)). The smallest
peak at 120 Hz is the result of the periodic foundation ((72.22 m/s)/0.6 m \( \approx
120.4 \text{ Hz} \)). Thus, the smallest peak at 120 Hz can not be the reason of the rumbling.
It seems to be admissible to neglect the periodic foundation.

We assume that the non-circular profiles of the wheels can be expanded into
Fourier series with a finite number of non-zero Fourier coefficients. The non-
circularities are described by \( \varphi \)-dependent heights of the rims; for one rim we
have:

\[
(2.27) \quad h_{wr}(\varphi) = h_{wr0} + \sum_{n=-N}^{N} C_{wnr} e^{jnp},
\]
where $C_{wn}$ are the complex Fourier coefficients; the coefficient $C_{wn}$ is the complex conjugate of $C_{wn}^{(\pi)}$. Since the model is linear, the forced oscillations caused by a single term $C_{wn}e^{jm\varphi}$ of the Fourier series can be calculated and the superposition of all terms yields the oscillations of the system. In the following we sketch how the oscillations caused by a single term are calculated, i.e. how the solution of the set of governing equations is determined.

For this harmonic excitation we write $u_r = \hat{u}_r e^{j\omega t}$ ($\omega = n\Omega$, $\Omega$ is the mean angular velocity of the wheel set, $j$ is the imaginary unit) and we consider this excitation as the right-hand side. First we separate the time dependence $e^{j\omega t}$. As a result, the partial differential equations which describe the one-dimensional continua rails and axle are simplified to ordinary differential equations with respect to the spatial variables $s$ or $y$, respectively, which are fulfilled by an $e^{kr}$- or an $e^{kya}$-ansatz. This approach leads to standard eigenvalue problems for the eigenvalues $\kappa_r$ and $\kappa_{ya}$, which are solved numerically. To solve the partial differential equations of the wheel disks, we separate the variables $\varphi$ and $r$ and choose a polynomial in $r$ for the $r$-dependence and an $e^{kr\varphi}$-ansatz for the $\varphi$-dependence. This procedure results in a standard eigenvalue problem for the eigenvalue $\kappa_w$. It is solved numerically, too. The numerically calculated solutions of the eigenvalue problems are substituted into the contact equations which leads to a set of linear algebraic equations. The coefficients are comprised in a matrix $M(\omega)$. We obtain the forced vibration of the system by inversion of the matrix $M(\omega)$ and multiplying the inverse by $\hat{u}_r$. Since we assume the model to be asymptotically stable (in the sense of Ljapunow), the inverse $M^{-1}(\omega)$ exists. One can find a detailed description of the procedure of solution in MEYWERK and BROMMUNDT [5] or MEYWERK [4].

In the following we sketch how the non-circular wheels force the system to oscillate. We start with the terms of excitation (the above mentioned right-hand-side-terms $u_r = \hat{u}_r e^{j\omega t}$) and concentrate on the terms caused by one wheel; the calculation for the second one has to be done in the same way. One excitation term is due to the vertical contact force $F_{c3}$ between wheel 1 and rail 1. It is:

\[
F_{c3} = P + k_H \underbrace{(f_w - f_r + h_{wr} - h_{wr0})}_{\Delta f}.
\]

(2.28)

Here $k_H$ is the linearized Hertzian contact stiffness and $P$ is the static load caused by the weight of the coach. The sum $\Delta f$ comprises the time-dependent elastic flattening of the wheel and the rail in the contact area. This flattening depends on the deflections and deformations of the wheel set $(u_{wa}, v_{wa}, w_{wa}, \alpha_{wa}, \beta_{wa}, \gamma_{wa}, u_{wh}, v_{wh}, w_{wh}, \alpha_{wh}, \beta_{wh}, \gamma_{wh}, v_{wd}, u_{wd}, w_{wd}, \text{abbreviated by } f_w)$, the rail $(u_r, \nu_r, w_r, \alpha_r, \beta_r, \gamma_r, \text{ abbreviated by } f_r)$, and on the difference between the mean value of the height of the rim $h_{wr0}$ and the height $h_{wr}$ of the non-circular wheel rim.
at the point of contact. Another excitation is due to the geometrical requirement that the tangential planes of the wheel and the rail should be parallel.

The height of the rim at the point of contact \( \tilde{h}_{wr} \), which enters Eq. (2.28), is in linear approximation \( \tilde{h}_{wr} = h_{wr}(\Omega t) \), where \( \Omega \) is the mean value of the angular velocity of the wheel set. We have

\[
F_{c3} = P + k_H \left( f_w - f_r + \sum_{n=-N}^{N} \frac{C_{wren}e^{in\Omega t}}{n\neq 0} \right),
\]

where we can identify the right-hand side excitation terms.

### 3. Numerical results

Measurements show that the rumbling is caused by non-circular wheels, i.e. rumbling can only be observed if the wheels are non-circular. The measured Fourier decomposition of the non-circularities of the wheels show that the absolute values of the Fourier coefficients have a maximum around the coefficient of the fifth harmonic (cf. Müller and Diener [6]).

For a better understanding of the growth of rumbling, it is helpful to look at the eigenvalues and the corresponding natural modes of the system. In Fig. 6 the eigenvalues of the system up to 150 Hz are depicted; the foundation is as stiff as it is at the new tracks of DB. We focus on the eigenvalues \( \lambda_4 \) and \( \lambda_5 \) which play the essential role in the growth of rumbling. First of all we have a look at the natural modes, which are depicted in Fig. 7 and Fig. 8. In these figures the undeformed wheel set and the undeformed tracks are outlined using thin circles or lines. The deformed ones are depicted by bold lines. To get a better impression of the deformation of the different parts, the deformed and the undeformed wheels, rails, and axle are joined by lines. The modes describe vertical oscillations of the wheels and the rails; in Fig. 7 the right and the left wheel-rail-pair oscillate in phase and in Fig. 8 in antiphase. These eigenmodes are mainly excited by non-circular wheels if the Fourier decomposition of the non-circularities contain the fourth, the fifth or the sixth harmonic and the velocity is approximately 60 m/s–70 m/s. Trains with such non-circular wheels travel both on new, stiff and on old, soft tracks, and the eigenmodes exist both on the new and old tracks.
Therefore, three questions arise:

1. Why does the rumbling occur on the new stiff tracks independently of the velocity of the trains?

2. Why does the rumbling not occur as often on the old tracks as it does on the new ones?

3. Why is the rumbling more noisy on the new tracks than it is on the older ones?
To answer the questions, we first look at the change of the eigenvalues $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_5$ if the vertical track stiffness $k_{rw}$ and damping $b_{rw}$ increases. The values $k_{rw}$ and $b_{rw}$ are the parameters of a Winkler foundation, i.e. the force per length in vertical direction, which is due to the foundation, is $b_{rw} \dot{w}_r + k_{rw} w_r$. The imaginary parts of the eigenvalues for the variation of $k_{rw}$ and $b_{rw}$ are depicted in Fig. 9. For the soft tracks one can see that the imaginary parts of those eigenvalues, which belong to the mainly vertical oscillations, are close together and at low frequencies (about 70 Hz). For the stiffer tracks the imaginary parts diverge and the frequencies increase (about 110 Hz). The small distance between the eigenvalues for the soft track and a larger distance for the stiffer track answer the first and the second question: A small distance between the eigenvalues causes a narrow resonance peak. For high vertical forces and loud rumbling it is necessary that the excitation from the wheels should fit the resonance peak. Therefore, the rumbling occurs in small velocity intervals only, i.e. in those velocity intervals in which the excitation frequency fits the resonance peak. A larger distance between the eigenvalues for the stiff tracks causes a wider peak. This raises the probability, that the frequency caused either by the fourth, the fifth, or by the sixth harmonic, fits the resonance peak. This means that an excitation frequency exists which fits the peak nearly independently of the velocity.

Higher frequencies of the mainly vertical oscillations (eigenmodes $\lambda_4$ and $\lambda_5$) partially answer the question three: The damper between bogie and coach (the so-called secondary suspension) is stiffer for higher frequencies. Thus, the oscillations of the wheel sets on the new stiffer tracks are not as well isolated from the coaches as on the softer tracks. But there is another reason for the noisy rumbling on the stiff tracks. We explain this reason with the help of Fig. 10. We
Fig. 9. Imaginary parts of the eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ as functions of the vertical stiffness parameters $k_{rw}, b_{rw}$.

Fig. 10. Vertical force $F_0$ as a function of the order of the Fourier expansion for different foundation parameters.

[329]
investigate a wheel set with a non-circular wheel 1 and choose the height $h_{wr}$ of the rim of wheel 1 to be:

$$h_{wr} = 20 \mu m \cos(n\varphi),$$

where $n$ is a natural number. This results in an excitation frequency of $\omega = n\Omega$. For this wheel set, the force $\bar{F}_b = F_b \cos(n\Omega + \alpha_s)$ ($\alpha_s$ phase shift) between the bogie and primary suspension is calculated; in Fig. 10 the amplitudes $F_b$ are shown. To obtain the resonance curves, the number $n$ is varied continuously. Of course, the values of $F_b$ for non-natural numbers are artificial, but they make sense for other velocities. The resonance peaks for the soft track (curve 1) and the stiff track (curve 5) differ in their height and width: The peak for the stiff track is higher and wider than the peak of the soft track. The consequences of the wider peak at a higher frequency have been discussed above. The height of the peak, which is larger for the stiff track than it is for the softer ones, increases the rumbling, additionally to the increase caused by the stiffer secondary suspension.

4. CONCLUSIONS

Some hints concerning the reasons of the rumbling in the high-speed train ICE of the DB are given. The resonance of two mainly vertical oscillating eigenmodes play the main part in the growth of rumbling. These eigenmodes are excited by non-circular wheels. The divergence and the increase of the imaginary parts of the corresponding eigenvalues, and higher forces between wheel set and bogie are the reasons for the frequent occurrence of rumbling on the new stiff tracks. Nevertheless, the growth of the non-circularities of the wheels is not explained yet.

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