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Changes in Strength Properties of Torsion-Loaded Thin-Walled Channel Beam Caused by Constructional Modifications Introduced Based on Statical Analyses

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The paper presents results of linear finite element method (FEM) analyses of a thin-walled channel beam in whose central part there are introduced some variants of constructional modifications aimed at improving beam's ability to carry torsional load. The obtained results show how radically the strength properties of thin-walled structures can be improved by applying simple constructional modifications designed on the basis of the so-called statical analyses. Solutions presented here were taken from the works [2, 5].

Statical analyses allow for finding solutions to constructional problems of complex thin-walled systems, e.g., the systems made of plane elements, well suited to the applied load [2, 5]. They also make it possible to identify cardinal errors in that class of structures and indicate methods of error elimination. Although methods of statical analysis have been studied for a long time, they are still not very popular despite the fact that to use them one only needs to know the fundamentals of statics.

The considered problem is important for engineering practice, because the thin-walled channel beams, which are commonly used, exhibit low rigidity to the assumed torsional load.

Key words: thin-walled structures, design, FEM analyses, statical analyses.

1. Introduction

Thin-walled structures, e.g., the ones made of plane elements, are commonly applied in systems used in building, automotive and aviation engineering. Thus, the task of designing such systems is in the center of engineers' attention.

The basic strength properties of such systems are determined already in the initial phase of design, and the key question is to correctly choose their structure for load they carry [2]. As a structure, we mean the number and spatial allocation of component elements and the system of mutual connections between them [2]. The structure must be selected in the way ensuring the possibility of carrying the applied forces by means of membrane states of stress, i.e., maintaining plane state of stress in each component element [2]. Such a kind of stress gives the possibility of taking full advantage of carrying capacities of the material, which is achieved thanks to uniform distribution of stresses across the thickness of elements' walls.

The work presents the results of linear FEM calculations for a series of thin-walled constructions, which were based on thin-walled channel beam, and whose structures were designed with the use of the so-called statical analyses to carry load applied by a torsional moment.

The main objective of this work is to show how radically one can improve load capacity by applying simple constructional solutions, obtained based on uncomplicated analyses, which can be easily performed by any mechanical engineer – because the only thing one should know are the fundamentals of statics.

Constructional schemes of the analysed FEM solutions are presented in Figs. 1b–d [2, 5]. The applied constructional means consisted in mounting welded

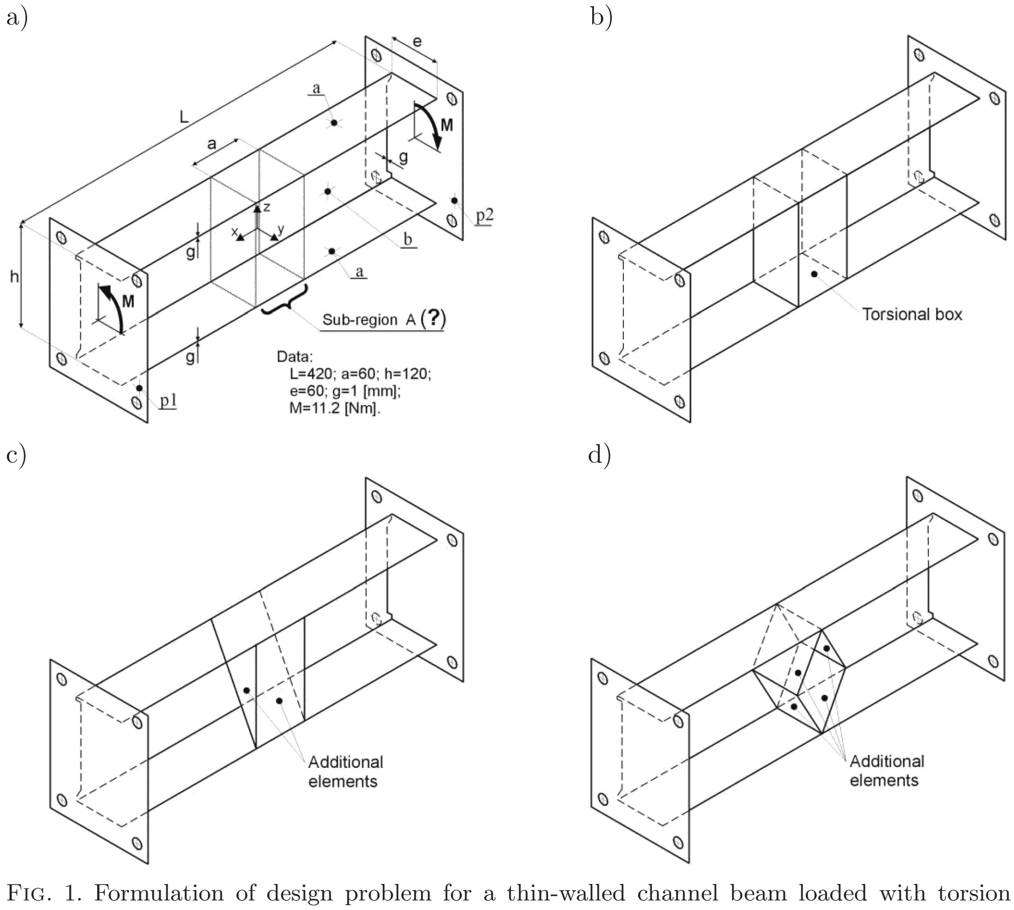

Fig. 1. Formulation of design problem for a thin-walled channel beam loaded with torsion moment and selected solutions analysed with FEM.

elements in the central part of the section: a torsional box (Fig. 1b), two additional plane elements (Fig. 1c) and a pipe of axis perpendicular to the section's axis (Fig. 1d).

These structures can be parts of more complex systems, and after additional modifications of their geometry they can even constitute standards for, e.g., cross-bars in vehicle frame bearers that carry torsional moment load.

In this work, we did not investigate the influence of shapes and dimensions of component elements on strength properties. These parameters can be determined, e.g., by using the method of statically admissible discontinuous stress field (the SADSF method, [2, 3]), or methods of shape optimization [1, 6, 8]. The structures from Figs. 1c and 1d, which are fully shaped by means of the SADSF method, can be found in the monograph [2]. FEM analyses for the structure shown in Fig. 1c are presented in [7].

Statical analyses, briefly described in the final part of the paper, are still very rarely used by engineers, however, they deserve popularization.

These analyses make it possible not only to solve the problems consisting in selecting structures for assumed forces, but also to verify the applicability of existing structures for the load they carry [2]. The mentioned analyses should be performed every time before FEM calculations, because they allow for identifying incorrect solutions of inferior load capacities, which are not worth being subjected to the calculations.

The formulation of the solution-seeking problem is presented in Fig. 1a [2]. The data are external loads, having the form of two opposite moments M, which are applied to two additional diaphragms, 'p1' and 'p2'. The task is to find such a structure of additional elements that the applied load could be carried in the membrane state. It is also assumed that all additional elements will be introduced in the central part of the section comprising the sub-region A of length a.

As one can see, the problem formulated in such a way has many possible solutions, and the number and quality of them depends on experience and inventiveness of an engineer.

Statical analyses were initiated by Brzoska [5] and recollected and continued by Bodaszewski, who substantiated applicability of the analyses on the basis of the theorems of statically admissible stress fields [2].

To date, the titles on statical analyses have only been published in Polish literature [2–4]. The present work is probably the first one which shows the great benefit that can result from applying these analyses.

The problems presented in this work are close to those considered within the framework of topology optimization methods [1, 6, 8], where one optimizes location of material within a specific area of the construction having assumed boundary conditions. Unfortunately, these methods need complicated and costly software, and are based on iterative approach, which is unsuitable for the considered class of structures in which even small change in structural parameters can cause radical deterioration of strength properties [2]. Then, those methods may lead to solutions that consist in attaining a local minimum; thus, such methods must be applied with great caution.

2. FEM calculational model

The analyses were carried out with the use of the FEM, utilizing the CosmosM software package. One assumed, among other things, [7]:

- linearly-elastic physical model of the material and small deformations,
- triangular shell elements with six nodes and six degrees of freedom in the node type SHELL6,
- thickness of all elements equal to 1 mm, and average dimension of finite elements equal to 5 mm,
- torsional moment load acting on front diaphragms, resulting from forces F_y applied to nodes lying on the borders of holes; in rear diaphragms, the nodes lying on the borders of holes were deprived of the degrees of freedom U_y, additionally, in order to prevent the possibility of rigid motion, fixed dislocations U_x, U_z and rotations R_y, R_z of nodes lying at the center of these diaphragms were assumed,
- the same value of load for all cases, equal to $M = 11.2$ [Nm].

The applied shell model allows for only approximate analysis of local three-dimensional states arising in the regions of joints between the plane component elements. To analyse these states, one needs to carry out separate, more detailed investigations with the use of a solid model.

Approximate rigidity of each model was determined from the formula: $k = M/\phi$, where M – assumed value of torsional moment, and ϕ – angle of rotation of upper border of diaphragm 'p1' (see Fig. 2a), calculated based on displacement values of its extreme nodes.

3. Results of analyses

The constructional scheme and boundary conditions assumed for the regular thin-walled channel beam are presented in Fig. 2a. Because this model has no ability to carry the applied load through membrane states (see Fig. 6a), the load must be transmitted through bending states, which generally are characterized by high level of stress intensity.

This is confirmed by the results of analyses. The equivalent stress originating from membrane state is close to zero in the whole volume of the struc-

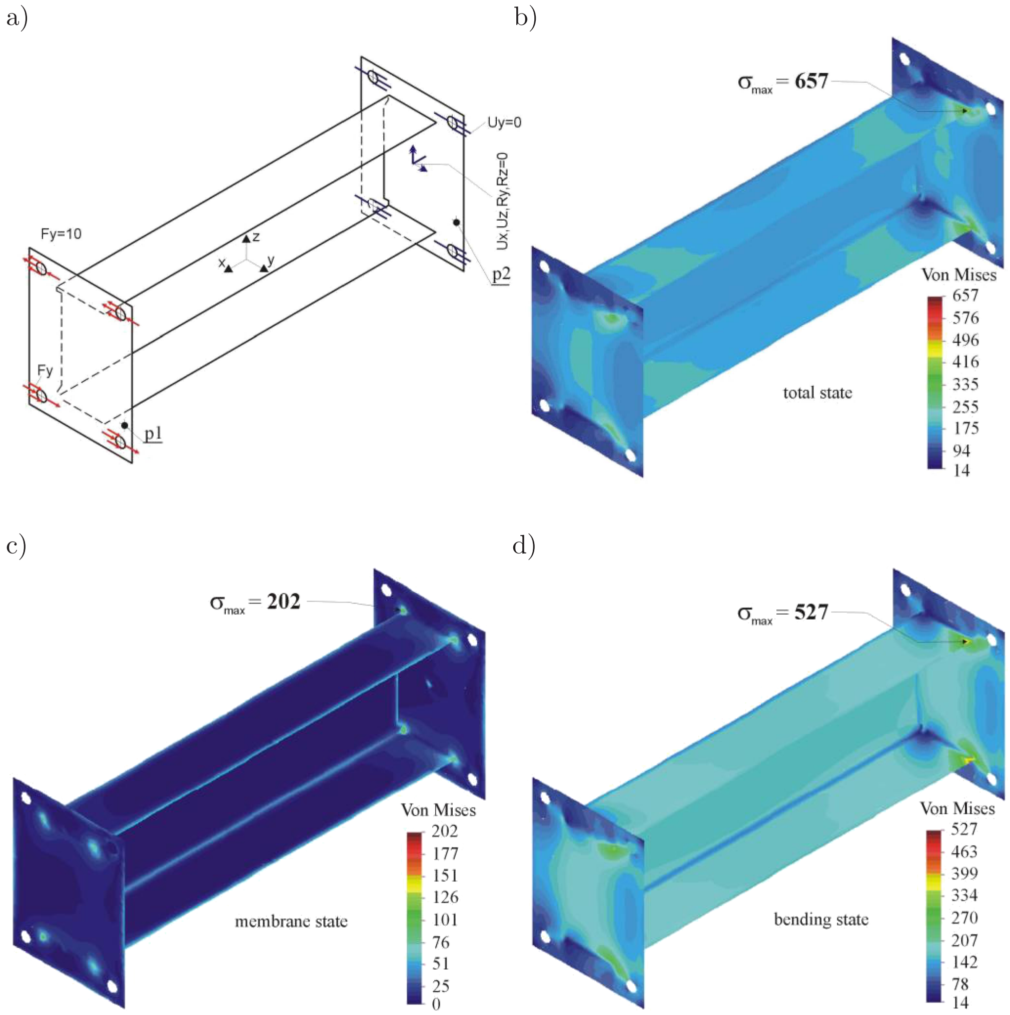

FIG. 2. Constructional scheme of model of regular thin-walled channel beam, boundary conditions assumed for FEM calculations and distributions of equivalent stresses calculated according to Huber-Misses criterion.

ture, only locally it reaches 202 MPa (Fig. 2c). The equivalent stress resulting from bending state is more diversified, however, its mean value equals approx. 180 MPa, and locally it reaches even 527 MPa (Fig. 2d). The highest equivalent stress resulting from bending is then more than 2.6 times greater than the highest equivalent stress associated with membrane state (527/202∼2.6). The distribution of total stress (Fig. 2b) is approximately equivalent to that resulting from bending states. The highest total equivalent stress takes the value of 657 MPa. The highest values of membrane and bending states appear in the vicinity of diaphragms, which is due to the fact that some small bimoment is transmitted to the diaphragms from the flanges within the limits of flexible rigidity.

Rigidity of this model equals: $k \approx 4.4$ [Nm/deg].

The results obtained for the thin-walled channel beam reinforced with a torsional box are shown in Fig. 3. This structure has the ability to carry the applied load in the membrane state (see Figs. 6b and 6c), which is demonstrated by domination of this state as well as radically better strength properties. The level of maximal total equivalent stress (Fig. 3a) has decreased as much as 14 times, com-

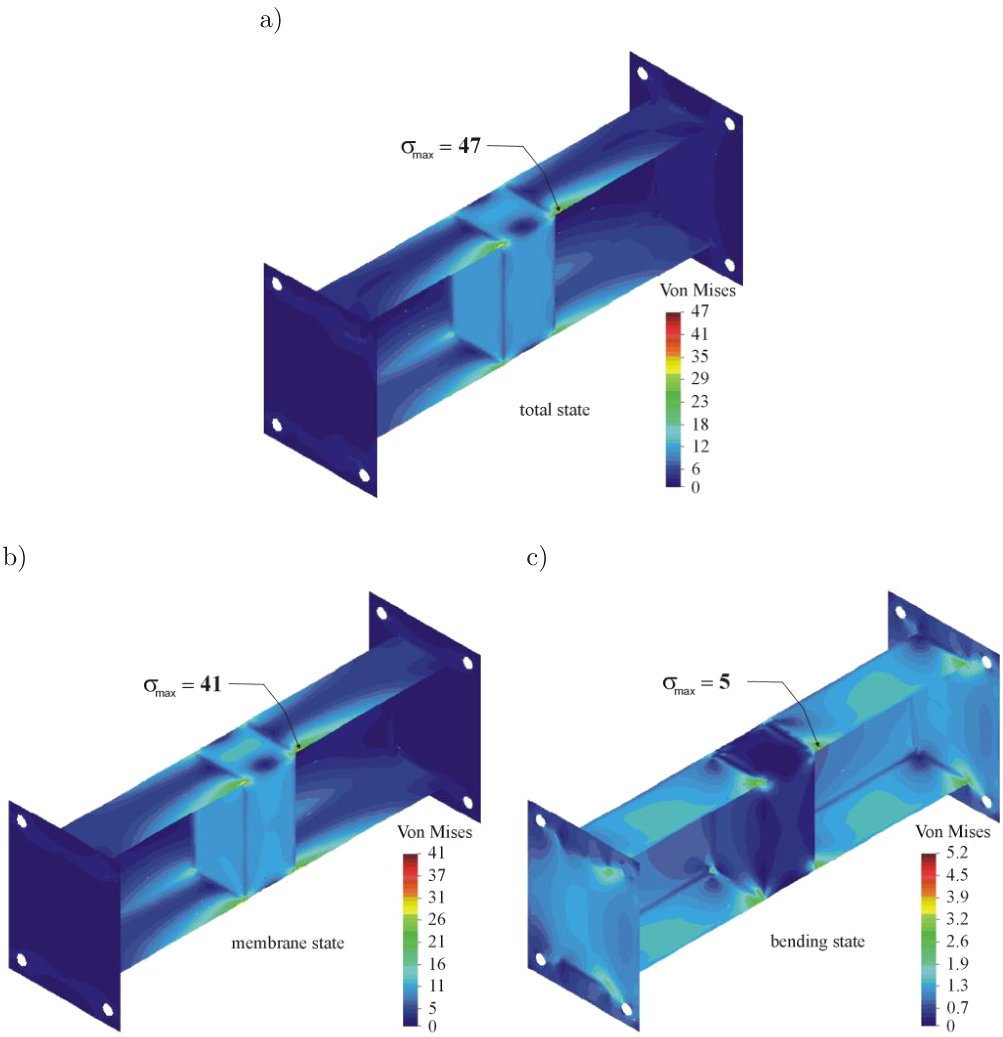

FIG. 3. Results of FEM analyses for the model of thin-walled channel beam reinforced with a torsional box.

pared to that in the model of regular thin-walled channel beam (657/47∼14). It means that, in order to obtain the same level of stress in the model of regular thin-walled channel beam, one should apply the load 14 times lower than that in the model reinforced by a torsional box. The greatest value of equivalent stress resulting from bending state (Fig. 3c) reaches here 5 MPa, which amounts to only approx. 12% (5/41∼0.12) of the maximal value obtained for membrane state (Fig. 3b). This time, the distribution of total equivalent stress is similar to that of membrane state.

The greatest total equivalent stresses appear in the flanges, which are bent in their planes, as it follows from statical analyses (see Fig. 6b). In the FEM image, one can see formation of bending axes in these elements, and the increase of bending moments in the direction from the diaphragms toward the torsional box. In the elements of torsional box, where pure shear is assumed, stress distributions are very well equalized. There are no significant loads on the web elements, for which it was presumed that no load would be carried. Among the solutions presented in this work, this is the one that has the best strength properties.

Rigidity of this model equals $k \approx 834$ [Nm/deg] and is as much as 190 times greater (834/4.4∼190) than that of the regular thin-walled channel beam.

The results obtained for the structures from Figs. 1c and 1d are presented in Figs. 4 and 5, respectively. Taking into account the systems of integral forces for these solutions [2], we may conclude that these cases are not very beneficial in terms of strength properties. This is because all their elements (with the exception of unloaded webs) are kinds of membranes bent in their planes. Then,

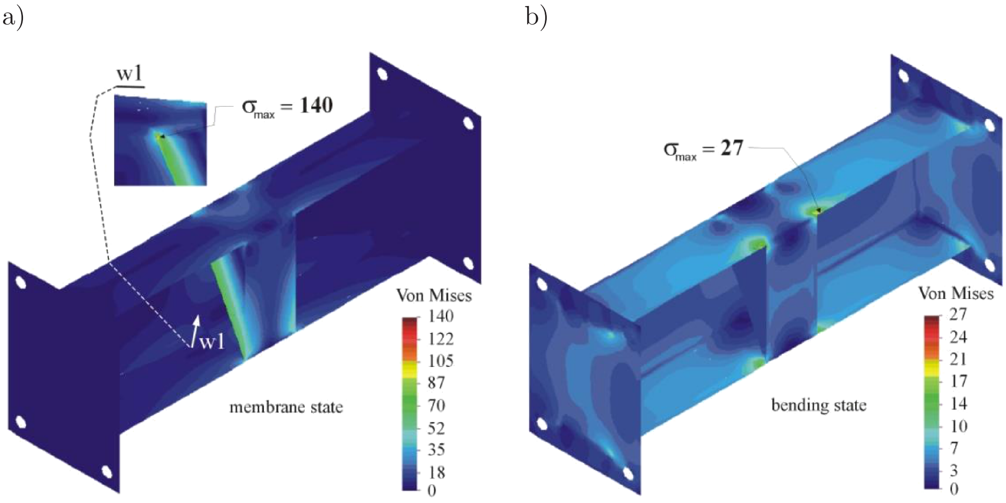

FIG. 4. Results of FEM analyses for the model of solution from Fig. 1c.

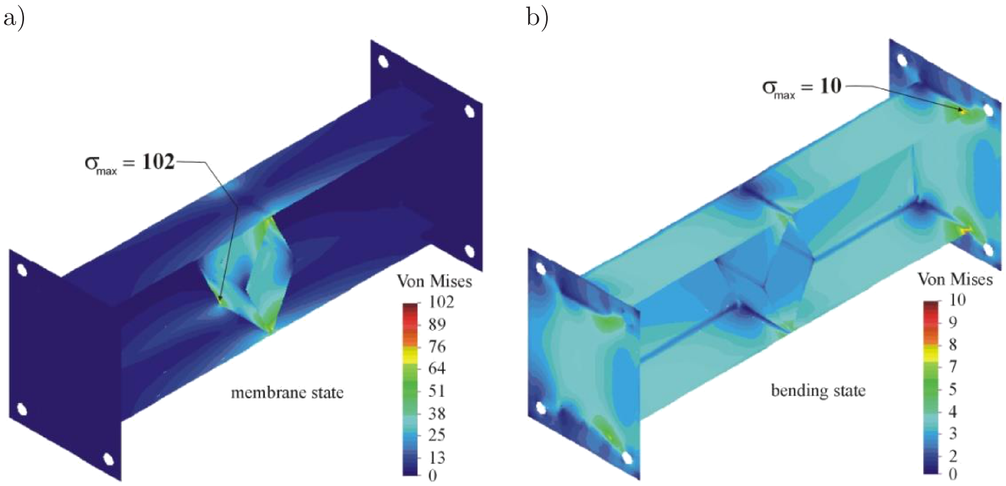

Fig. 5. Results of FEM analyses for the model of solution from Fig. 1d.

a bending axis appears in these elements, and the stress increases with the distance from this axis. Despite this fact, the maximal level of total equivalent stress is, in both cases, several times lower compared to that in the model of regular thin-walled channel beam. In the case of structure from Fig. 4 it is almost five times lower (657/141∼4.7), and in the structure from Fig. 5 – over six times smaller (657/104∼6.3).

The level of stress originating from bending is low. In the case of structure from Fig. 4 the greatest equivalent stress related to bending amounts to 19% (27/140∼0.19), and in the case of structure from Fig. 5 to barely 10% (10/102∼0.1) of maximal stress associated with membrane state. The distributions of total equivalent stress are very similar to the distributions for membrane states, and therefore are not presented in the figures.

Rigidities of the models were equal to $k \approx 196$ [Nm/deg] for the model from Fig. 4, and $k \approx 237$ [Nm/deg] for the second one. Then, they were greater 44 and 55 times, respectively, than rigidity of the model of regular thin-walled channel beam.

A further improvement of strength properties can be achieved by introducing corrections in shape and thickness of component elements, first in all of those in which the equivalent stress takes the greatest value. Such a task is beyond the scope of this study. However, it is worth noting that after, e.g., two-fold increase in thickness of the additional oblique element from Fig. 4, one can achieve a decrease in maximal equivalent stress from 141 to 93 MPa.

Contrary to the structural corrections, the corrections of geometry do not cause such significant changes, neither in the character of work of the structure, nor in the level of equivalent stress.

4. Examples of statical analyses for regular thin-walled channel beam and thin-walled channel beam reinforced with torsional box [2]

In order to clarify the method of performing statical analyses, we present examples of such analyses for a regular thin-walled channel beam and a beam reinforced with a torsional box (Fig. 1b). Full information about this method, and many interesting examples, including those presented in this paper, can be found in the recently published monograph [2].

One begins statical analyses from disassembling the structure into plane component elements. Next, one applies to each of these elements integral membrane forces originating from external load and reactions of adjacent elements (Fig. 6). To prove the possibility of existence of a membrane state, it is enough to detect at least one system of forces which satisfies equilibrium equation for each of the elements [2]. If such equilibrium is not possible, it means that the structure carries load through bending forces.

The moments M are introduced into the elements of flanges by applying the forces P, which create the pairs $M = Ph$. The two diaphragms, 'p1' and 'p2', are used for this purpose (Fig. 6).

In the case of regular thin-walled channel beam (Fig. 6a) it is not possible to attain equilibrium for flanges loaded with forces P, creating the pair PL, by means of membrane forces only. Equilibrium cannot be ensured by forces T acting on common borders between the flanges and the web, which – as it follows from the web's equilibrium equations – must take zero values. In the figure above, all the forces that cannot realize themselves are marked with crosses. As one can see, this structure will carry the load M through bending states.

It is worth noting that, in order to ensure equilibrium for the flanges, one must insert between them a system of elements, which should carry the torsional moment equal to PL. This task can be fulfilled, for example, by the torsional box (one could also use a pipe of circular cross-section [5]). Such a solution is presented in Fig. 1b, and its statics is illustrated in Figs. 6b and 6c. The moments PL acting on the flanges are balanced by the pairs of forces Ra and Te. As it can be seen in Fig 6c, each element of the torsional box carries only shear forces R, T and V.

Equilibrium equations for individual elements can be formulated as follows:

$$M = Ph; \qquad PL = Ra + Te; \qquad Th = Va; \qquad Rh = Ve.$$

Having the value of moment M, and the assumed dimensions of the structure, one can determine all the remaining forces:

$$P = \frac{M}{h}; \qquad T = P\frac{L}{e}; \qquad R = P\frac{L}{a}; \qquad V = P\frac{Lh}{ae}.$$

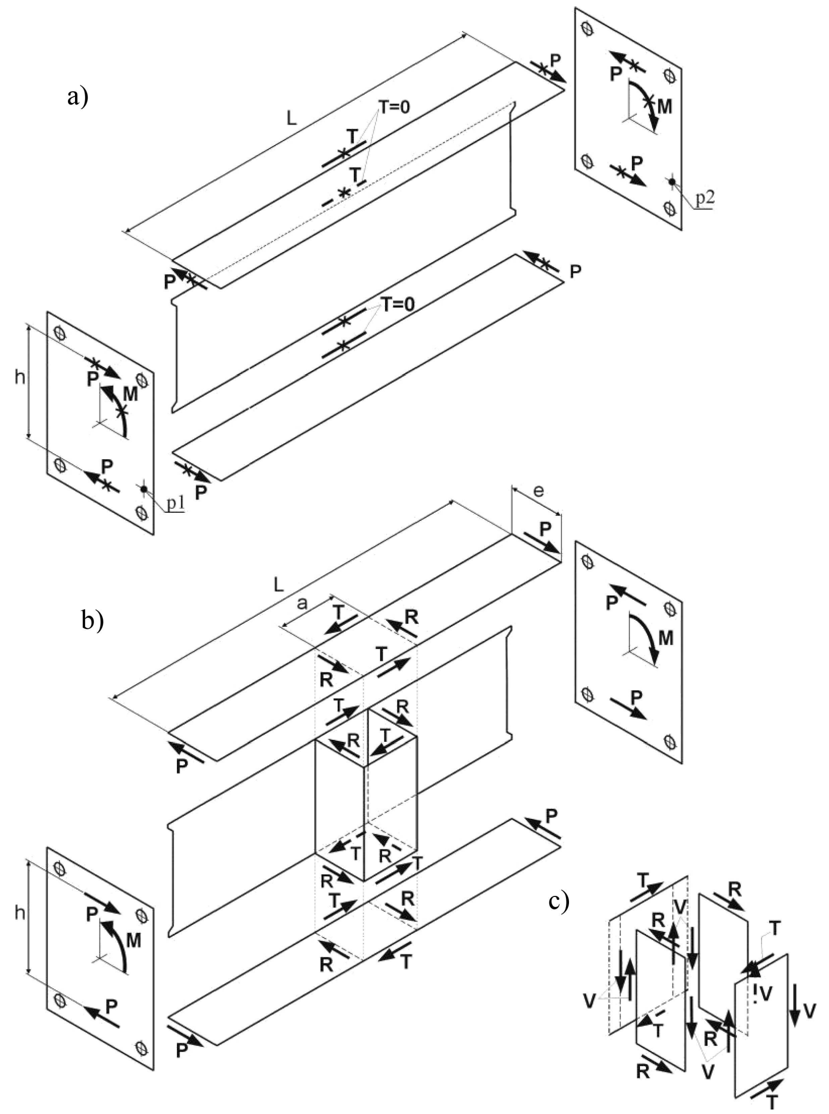

FIG. 6. Statical analyses for regular thin-walled channel beam and the beam reinforced with torsional box [2].

In the scheme of decomposition of the structure into component elements, presented in Fig. 6b, one neglects additional division of the flange and the web into elements for separation of the torsional box subarea. Such decomposition and the resulting system of forces are even more complicated, and are presented in detail in monograph [2, Fig. 3.23]. In statical analyses, it is enough to find only one statically admissible system of integral membrane forces. The system presented in Fig. 6b seems to be the simplest one.

Two variants of the statics for the solution from Fig. 1c one can find in the monographs [2, Fig. 3.25] and [3, Fig. 9.2].

5. Conclusions

Statical analyses allow us to draw only qualitative conclusions. However, they make it possible to step up by several times (by even more than a dozen times) the level of strength properties of thin-walled structures in comparison to the systems designed in a traditional way.

As one can see, the considered class of constructions is particularly sensitive to the errors in selection of structures. The finite element method allows for detecting these errors, but it becomes possible only after performing complete analyses, moreover, the FEM provides no hints about the direction of necessary changes [2].

A further improvement of strength properties of the analysed structures can be achieved through corrections of geometric parameters of component elements. The best way to do so is to apply the SADSF method [3], which makes it possible to determine them in a simple way, without using iterative procedures.

Using this method, one encounters a multitude of possible solutions, which can be utilized in quest for such solutions that satisfy some additional criteria, including, e.g., technological, economic and strength limitations, etc. Some of these criteria can be taken into account already at the stage of selecting the structure.

Besides modifications in region A, presented in this work, one can also consider many other variants. It can be shown that, in all cases, one achieves strength properties analogous to these presented here.

The results of FEM analyses, obtained in this work, allow us to assess not only the degree of improvement of strength properties in reference to the model of regular thin-walled channel beam, but also to evaluate the quality of each solution.

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