

METHODOLOGY FOR COMPARISON OF HYDROCODE MODELLING
WITH EXPERIMENT FOR SPLIT HOPKINSON PRESSURE BAR (SHPB)
TESTING OF SOFT MATERIALS

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The objective of this work is to develop a technique for reliable comparison of simulations with SHPB data in order to validate material models for “soft” materials such as polymers. Comparison with an output stress-strain curve is not sufficient since there are many assumptions built into this analysis. Primarily these concern the notion that the specimen is in stress equilibrium and volume is conserved. The problem is that the choice of material model for the specimen in the simulation dictates how and when the specimen attains stress equilibrium. The main methodology is based on comparing the simulations with the raw strain gauge data on the input and output bars, which makes no assumptions about stress equilibrium. However, one has to account for the well documented Pochhammer-Chree oscillations and their effect on the specimen.

1. INTRODUCTION

The Split Hopkinson Pressure Bar (SHPB) has been in use for many years, mainly for metals and also more recently for much softer materials such as polymer, usually in compression. The experimental technique is well documented through papers by KOLSKY [1] and there is an excellent review paper by GRAY *et al.* [2]. The SHPB test is regarded as the key validation test for material models at high strain rate. The main reason for this is that the specimen is considered to be in uniaxial stress after attaining stress equilibrium and high strains can be achieved through purely elastic loading in the bars where there are no shock waves present. Also for metals equilibrium is seen to be obtained

after three stress wave transits in the specimen based on one-wave versus three-wave analysis and the onset of plasticity ensuring no volume change in the specimen. Thus one simply needs to monitor the stress v strain response in the specimen to compare with the experimental output. For softer non-metallic materials such as polymers the deformation mechanisms are driven by elastic behaviour and thus it is not so clear-cut when stress equilibrium is obtained – if indeed the sample is ever in equilibrium during the test. In simulations the choice of material model is the prime determinant of whether the specimen reaches stress equilibrium.

QinetiQ has been involved in the development of physically-based material models for polymer composites, using basic polymer theory. There is a great need to have a reliable means of validating the models, particularly at high strain rates.

This paper aims to highlight the issues with the current techniques in terms of reliably comparing a simulation with an experiment. The paper then outlines a method for reliable comparison, which also sheds great insight into the SHPB test in general. Some recommendations are also made for future studies.

2. EXPERIMENTS

The Standard SHPB set-up consists of a striker bar hitting an input bar and an output bar with a specimen between the two bars, as shown in Fig. 1.

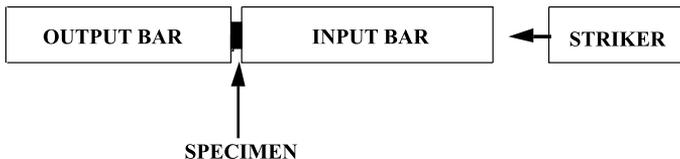


FIG. 1. Standard SHPB set-up.

The general technique is to use strain gauges on the input and output bars which are used to monitor the input, reflected and transmitted pulses. The standard 1D equations attributable to KOLSKY [1] are then applied to derive the stress and strain in the specimen. However, there are a number of fundamental assumptions in these equations, in particular related to the stress equilibrium in the specimen and no volume change in the specimen during the duration of the test. This analysis is also commonly referred to as a one-wave analysis, since it relies on the transmitted pulse to calculate the stress and strain in the sample.

A more sophisticated analysis is to use the so-called 3-wave analysis which uses the relation between the input, reflected and transmitted pulse and it is therefore a more complete analysis. However, even this assumes equilibrium and volume conservation in the specimen. To illustrate how complex these analyses

are a comparison of a 1-wave and a 3-wave analysis for a polymer composite is shown in Fig. 2.

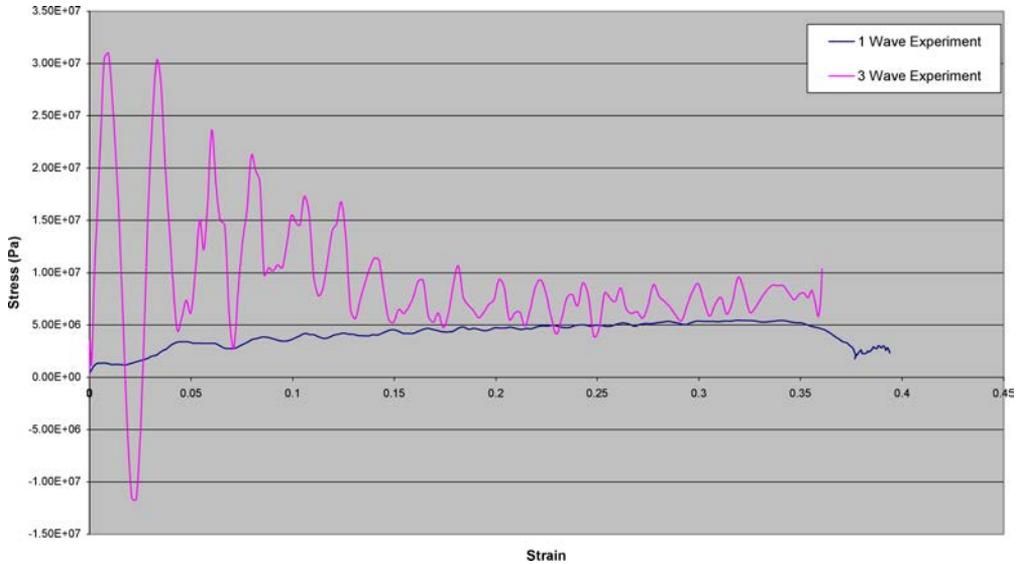


FIG. 2. Comparison of 1-wave and 3-wave analysis for polymer composite.

The assumption is made that the specimen is in equilibrium when the 1-wave and 3-wave analyses are similar. From the figure this is only true at relatively high strains above 0.2 and even then the judgment is subjective. The stress/strain trace is further complicated since there are significant oscillations, resulting from the Pochhammer-Chree (P-C) oscillations present when an elastic wave propagates down the bar. Therefore, it is very difficult to ascertain the true stress/strain behaviour in the sample.

The way to overcome this is to simply use the raw gauge data from the input and output bars for the incident, reflected and transmitted pulses directly. Thus, there are no implicit assumptions about the stress equilibrium or volume conservation in the specimen as the gauges only measured waves in elastic bars. One issue is that for soft materials the transmitted pulse is very small and thus the gauges have to be sufficiently sensitive to resolve these small stresses. This requires the use of semi-conductor gauges to ensure that the signal to noise ratio is sufficiently small. An example of the input, reflected and transmitted pulses for a polymer composite is shown in Fig. 3, where it is noted that the “noise” level on the traces is sufficiently low to enable the stress to be monitored.

The 1-wave and 3-wave analyses are still of value as these are the only means of comparing with an analytic model for the polymer composite material.

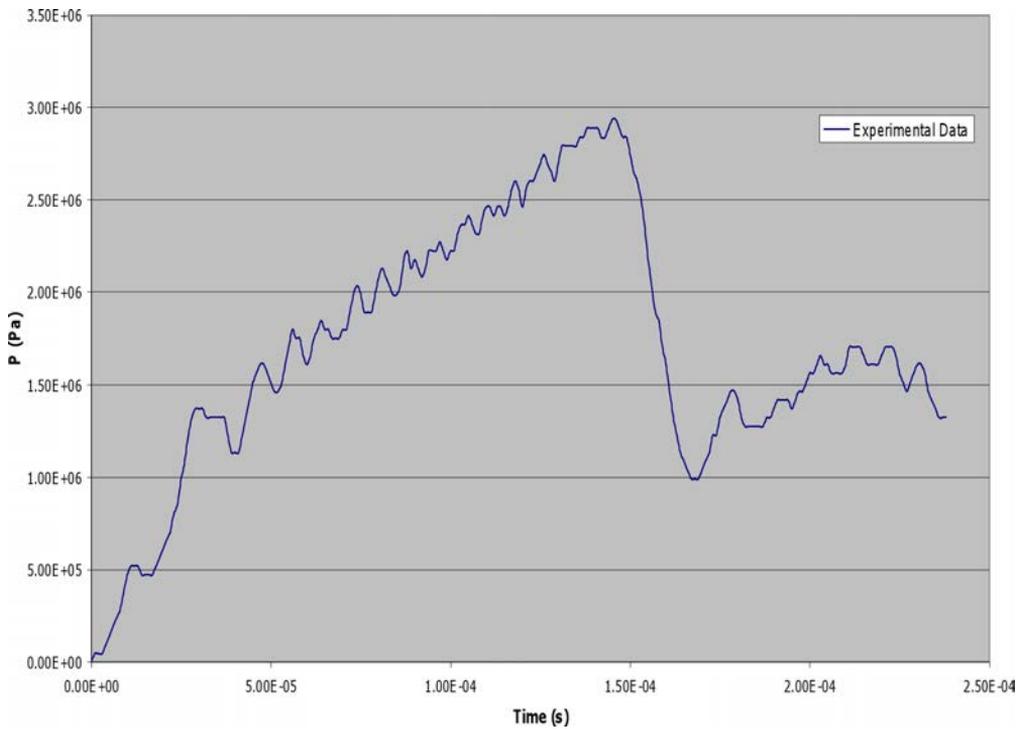
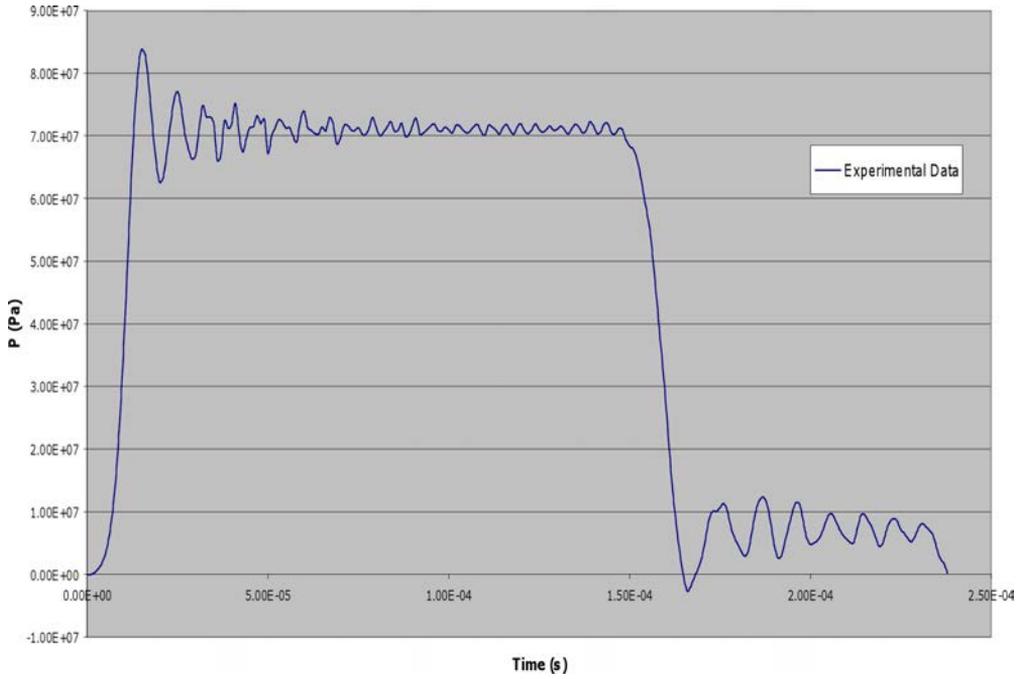


FIG. 3. Experimental input and output SHPB traces for a polymer composite.

3. MATERIAL MODEL

QinetiQ has recently developed the Porter Gould model for polymer composites which uses properties of the particles, the binder and the particle size distribution to determine properties of the composite as described by PORTER and CORNISH [3, 4]. The model assumes that the material follows Hooke's Law but that the secant modulus is a function of the damage in the composite. So the stress/strain curve for uniaxial tension or compression is generated by:

$$\sigma = E_c \varepsilon.$$

The composite, designated QRX221, comprises RDX explosive crystals in an HTPB binder. The particles in the composite range in size from below 1 μm to above 1 mm so we split the particle size distribution into four mass fractions: of order 1 μm , of order 10 μm , of order 100 μm and of order 1 mm. The particle size distribution is known so these mass fractions can be calculated.

The composite modulus, E_c , is calculated by considering each length scale in turn. It is assumed that, at the 1 μm length scale, the larger particles play no part in determining modulus. Thus, by denoting the particle modulus as E_p and binder modulus as E_b we calculate the composite modulus at this level as:

$$\frac{1}{E_{c,1}} = \frac{V_1}{E_p} + \frac{1 - V_1}{E_b},$$

where V_1 is the volume fraction of particles at this length scale.

At the 10 μm length scale we assume that the smaller length scale is sufficiently small that it appears as a continuum and so calculate modulus at this scale as:

$$\frac{1}{E_{c,10}} = \frac{V_2}{E_p} + \frac{1 - V_2}{E_{c,1}}.$$

Similarly at the 100 μm length scale:

$$\frac{1}{E_{c,100}} = \frac{V_3}{E_p} + \frac{1 - V_3}{E_{c,10}}.$$

Finally, the composite modulus for the entire material is:

$$\frac{1}{E_c} = \frac{V_4}{E_p} + \frac{1 - V_4}{E_{c,100}}.$$

Damage to the composite material is assumed to occur by a Griffith-type debonding process whereby work done to the material activates cracking in the binder. This cracking causes a loss of constraint and thereby reduces the modulus of the binder. We define two binder moduli, E_{dam} and E_{undam} , where E_{dam} is the damaged modulus and E_{undam} is the undamaged modulus. Each small local volume of binder is equally likely to crack and so we consider each to be a separate

state. The fraction of failed states – that is the fraction of small local volumes that have cracked and thereby have E_{dam} rather than E_{undam} – is denoted f . The binder modulus is therefore:

$$E_b = E_{\text{undam}} - f \cdot (E_{\text{undam}} - E_{\text{dam}}).$$

We now need to know the work done that the binder sees. This is a function of the compliances. The “binder” at the highest length scale sees a fraction of the work done on the composite:

$$W_{c,100} = \frac{E_p}{E_p + E_{c,100}} W_c$$

and so at lower length scales:

$$W_{c,10} = \frac{E_p}{E_p + E_{c,10}} W_{c,100},$$

$$W_{c,1} = \frac{E_p}{E_p + E_{c,1}} W_{c,10},$$

$$W_b = \frac{E_p}{E_p + E_b} W_{c,1}.$$

The probability of cracking is an activated process. If W_0 is an energy density characteristic of failure in the binder then the fraction of failed states is:

$$f = \frac{2 \cdot \exp(-W_0/W_b)}{1 + \exp(-W_0/W_b)}.$$

The model presumes that all temperature and strain-rate dependency is in the binder response as this dominates the mechanical response of the polymer composite. The model therefore uses a single particle shear modulus of 5.0×10^9 Pa, and a particle Poisson’s ratio of 0.3 under all conditions. The binder Poisson’s ratio is taken to be 0.4998. The particle size distribution is assumed to be as shown in Table 1.

Table 1. Particle size distribution for QRX221 composite.

lengthscale	V	$1 - V$
1	0	1
2	0.378	0.622
3	0.469	0.531
4	0	1

Implementation of the model into numerical schemes needs the full 3D behaviour and the shear modulus, G , is more appropriate than Young’s modulus. The above equations are still used, however, via the relation:

$$G = \frac{E}{2(1 + \nu)},$$

where ν is Poisson’s ratio. The constitutive model seeks predictions of G (undamaged), G (damaged) and W_0 for the binder as a function of strain-rate and temperature. All units are SI. The strain rate and temperature are first transformed via:

$$X = C_1 - C_2 \cdot [T - C_3 \cdot \ln(\dot{\epsilon})],$$

where T and $\dot{\epsilon}$ are temperature and strain rate and C_1, C_2, C_3 are constants as an attempt to mimic time-temperature superposition. The constants are chosen so that X increases with increasing rate and decreasing temperature and should always be positive for any condition likely to be found in reality. In practice, the data consists of temperature series at two different rates and the constants are chosen so that the two temperature series at the two rates form a continuous smooth curve.

The following equations are then used:

$$G_{\text{undamaged}} = A \cdot \exp [C_4 + C_5 \cdot \exp(X)] + S \cdot 9 \cdot \frac{1 - 2 \cdot \nu}{1 + \nu} \cdot P,$$

$$G_{\text{damaged}} = A \cdot \exp [C_6 + C_7 \cdot \exp(X)] + S \cdot 9 \cdot \frac{1 - 2 \cdot \nu}{1 + \nu} \cdot P,$$

$$E_0 = B \cdot \exp [C_8 + C_9 \cdot X].$$

The fitting constants for the QRX221 compositions are:

Table 2. Constants for Porter-Gould model for QRX221 composition.

constant	Value	unit
C_1	5	
C_2	0.01	K^{-1}
C_3	10	K
C_4	13.5	
C_5	0.14	
C_6	10.3	
C_7	0.21	
C_8	11	
C_9	0.45	

The constants A and B are not fitting constants but are used to convert G and E_0 to the correct units. For SI units $A = 1 \text{ Pa}$ and $B = 1 \text{ Jm}^{-3}$. S is a switch constant to turn pressure-dependency off or on and so takes values of either 0 or 1. P is the pressure and ν is Poisson's ratio = 0.4998. An example of the model comparison to SHPB data for the QRX221 is shown in Fig. 4.

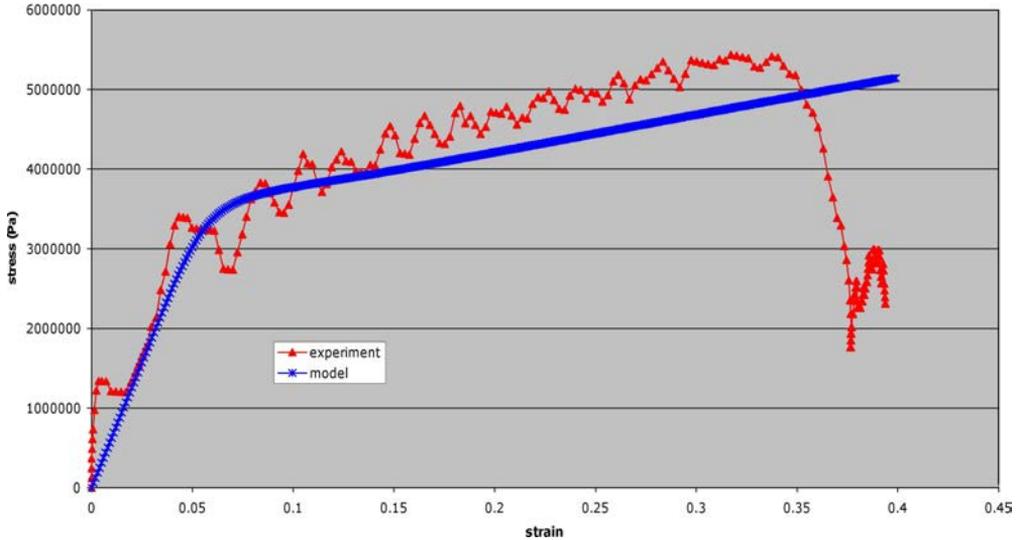


FIG. 4. Comparison of stress v strain from Porter-Gould model to SHPB test at room temperature for QRX221.

4. SIMULATIONS

The whole point of the SHPB test is that it is a high strain rate validation test for constitutive models in hydrocodes. As described above it is problematic to compare the hydrocode output directly with the stress/strain output from the experiment. The proposed route forward is to use the hydrocode to compare directly with the input, reflected and output gauge pulses. However, this implies that the hydrocode is capable of capturing all the physics in the SHPB in terms of the input, reflected and output pulses. This is actually quite complicated since the nature of the P-C oscillations must also be accounted for, since this produces a load/unload cycle in the specimen.

In terms of the input pulse, a typical comparison of simulation and experiment using the DYNA3D Lagrangian hydrocode is shown in Fig. 5. As can be seen the comparison is very good even in terms of the general P-C oscillations. This comparison is largely insensitive to artificial viscosity in the hydrocode.

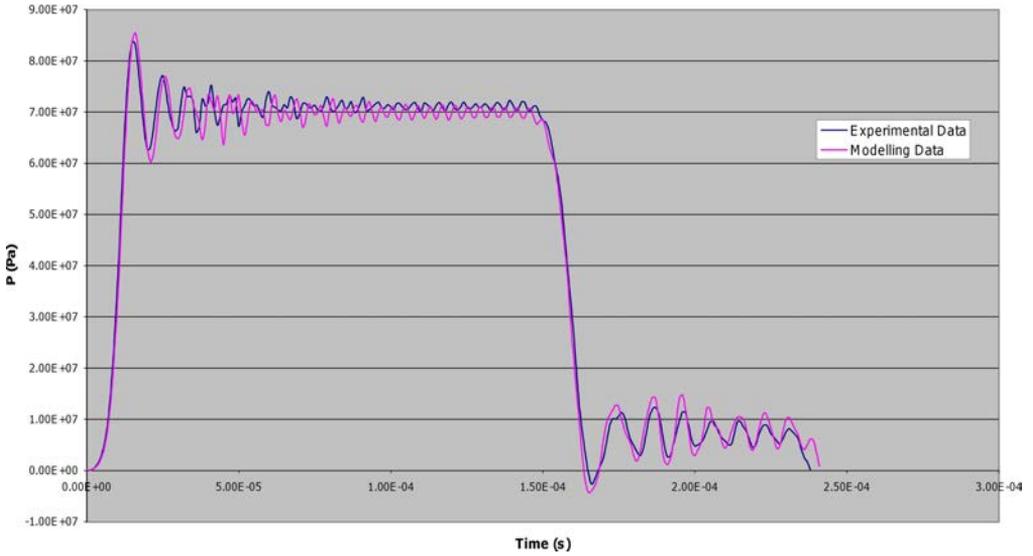


FIG. 5. Comparison of simulation and experiment for input pulses.

It is worth noting that the P-C oscillations can be removed by putting a curved impact face on the striker bar as shown in Fig. 6. This is useful

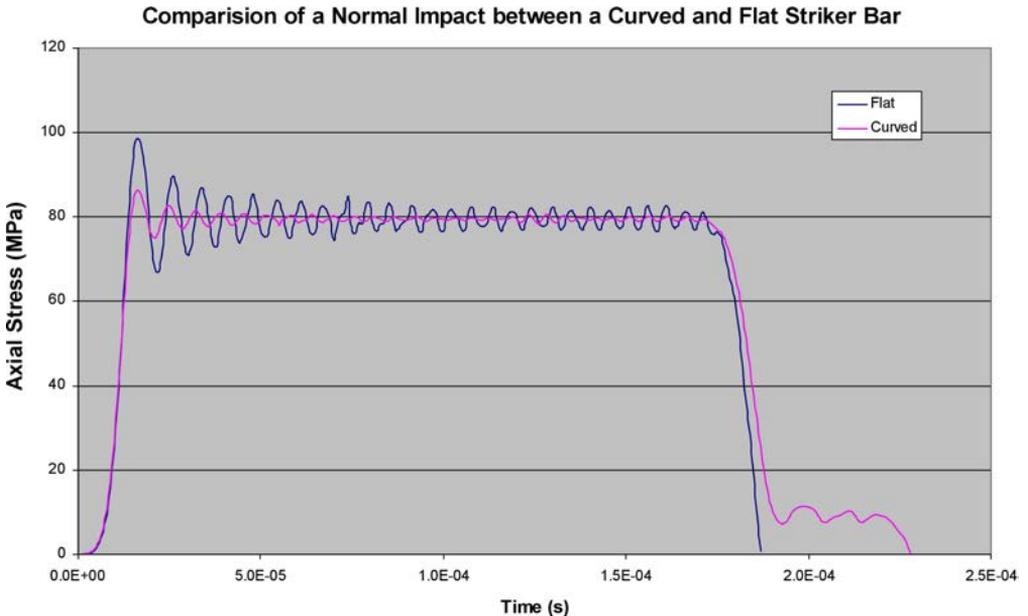


FIG. 6. Comparison of input pulses for flat and curved impact face on the striker bar for SHPB.

since it means that SHPB tests can be performed where the P-C oscillations are minimised and so the output can be compared directly with analytic models. Conversely, the P-C oscillations provide a load/unload cycle on the sample which is a very stringent test of the material model when implemented into the hydrocode.

The comparison between the hydrocode model and experiment for the output pulse is shown in Fig. 7, where there are no implicit assumptions concerning specimen equilibrium or volume conservation.

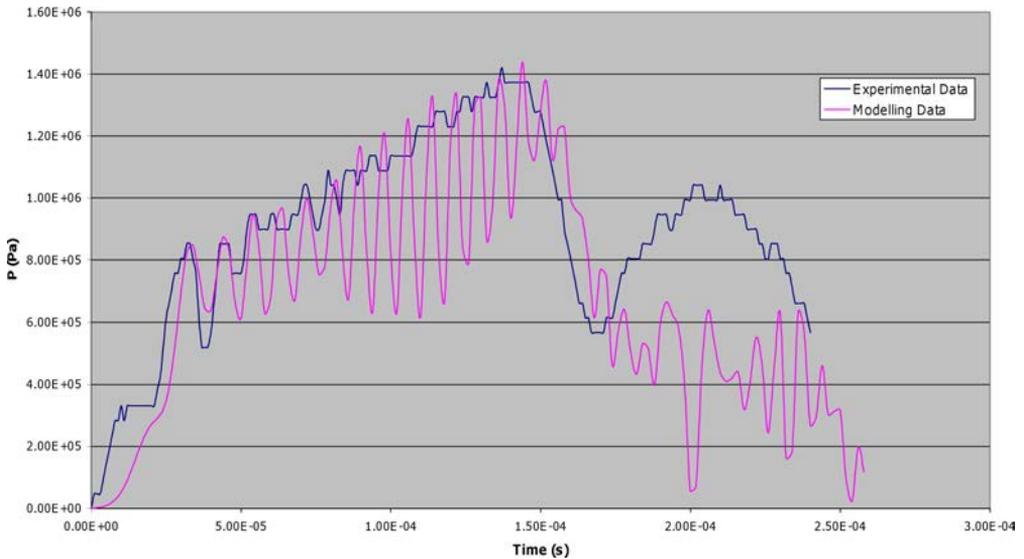


FIG. 7. Comparison of model against experiment for output pulse for QRX221.

The level of agreement for the general stress level and pulse length is good, although it is noted that the model exhibits significantly more oscillation than the experiment. The main reason for this is that the load/unload capability of the model is still limited and requires a true physically-based model for the polymer, which is being developed. However, without a robust method of comparing hydrocode with experiment it would be unclear where the deficiencies in the model lay. In addition the model does not pick up the very high initial modulus in the output pulse, which also requires further theoretical analysis. One can also still perform the 1-wave and 3-wave analyses on the pulses for comparison with experiment as shown in Fig. 8 for the 1-wave analysis.

To further demonstrate the power of the approach a comparison with an SHPB test performed at 223 K is shown in Fig. 9.

The level of agreement is very impressive in terms of the stress level. The length of the pulse is different since the specimen exhibited radial cracks during

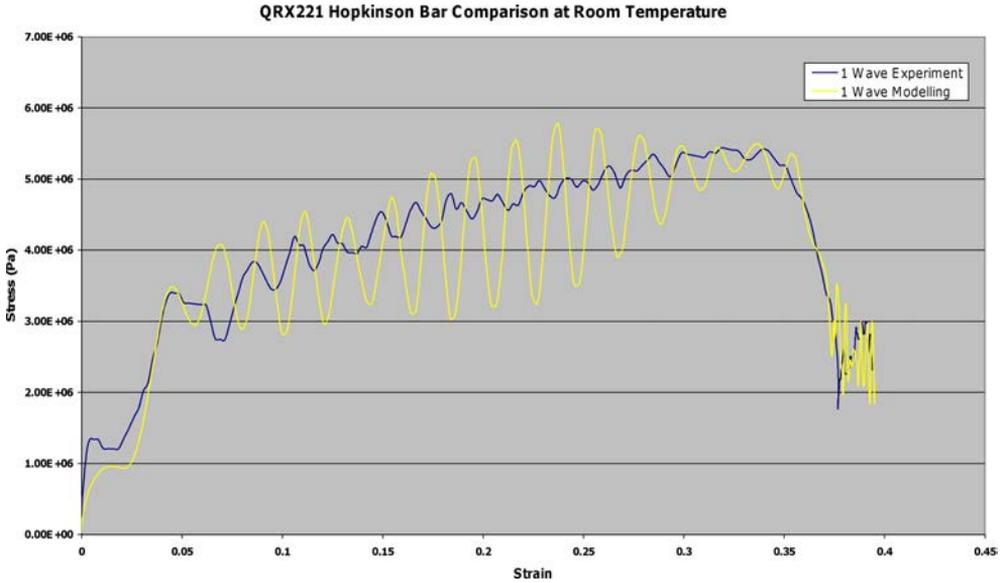


FIG. 8. Comparison of modelling and experiment for 1-wave analysis for QRX221.

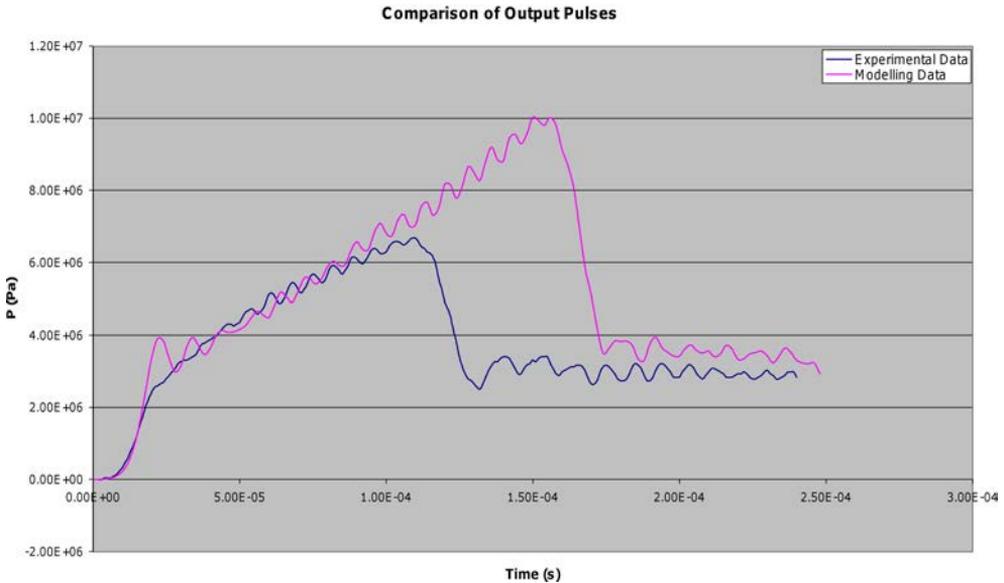


FIG. 9. Comparison of model and experiment for QRX221 at 223 K.

the test. Indeed the time of the pulse can be directly correlated to the timescale of the macroscopic fracture in the specimen. This general level of agreement gives enhanced confidence that the models are suitable for use in real applications such as hazard assessment of munitions, gun launch, etc.

5. CONCLUSIONS

1. A methodology for effective comparison of simulations with SHPB experiments has been defined based on comparison of the simulations with the raw gauge data. The methodology is robust for general “soft” materials over a range of temperatures.
2. This has been demonstrated for the QRX221 polymer composite and has highlighted further required developments to the model in the form of a physically based polymer model and better load/unload functionality.
3. The P-C oscillations are a good test of the material model as they provide a load/unload cycle in the specimen during the test.
4. The approach gives added confidence that the models can be effectively applied to real scenarios.

ACKNOWLEDGMENT

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